

Project 1: Review of Functions

Part 1: Factoring Polynomials

To do this part of the project, all we need to know is:

- What the different methods are for factoring a polynomial.

There are several different factoring methods, and one of the hardest parts of factoring is figuring out which one to use with a given polynomial. Here are the different methods you probably learned in earlier classes:

- Factoring out the greatest common factor
- Factoring by grouping
- Factoring a trinomial by trial and error
- Factoring a trinomial by using the “*ac* method”

(To factor $ax^2 + bx + c$, 1) Form the product ac ; 2) Find a pair of numbers whose product is ac and whose sum is b ; 3) Rewrite the polynomial to be factored so that middle term bx is written as the sum of two terms whose coefficients are the two numbers found in Step 2; and 4) Factor by grouping.)

- Factoring the difference of two squares
- Factoring the sum or difference of two cubes

Our goal on this part of the assignment: To be able to identify which factoring methods are best just by looking at a polynomial that needs to be factored (and then to be able to correctly apply those methods); to understand *why* these methods work.

First we want to practice identifying which of these methods should be necessary in order to factor a given polynomial, without even needing to factor the polynomial itself – we want to test if we can choose the correct method. So, for each of the following polynomials, identify ALL of the factoring methods needed to factor the polynomial COMPLETELY. (Sometimes there may be several different correct answers here, because sometimes more than one method works.) **Rotate roles for each number: For each problem, the *Prover* should factor the expression completely, and the *Explainer* should identify ALL the methods that have been used to factor each expression and explain each step that the *Prover* has taken to complete the factorization.**

1. $x^2 - 5x + 6$
2. $9 - 4x^2$
3. $x^2 - 18x + 81$
4. $3x^4 - 14x^2 - 5$
5. $x^2 + x + 9$
6. $x^4 - 16$
7. $2x^3 + 4x^2 - 30x$
8. $x^3y^3 + 5x^2y^3 + 2x^3 + 10x^2$
9. $x^2 + 49$
10. $x^3 - 8$

Extra Credit: (Only do this part AFTER you’ve finished Part II below!) Create your own factoring problems! Create a polynomial that satisfies each of the following criteria:

1. A factorable trinomial with a leading coefficient of 1
2. A factorable trinomial with a leading coefficient that is not 1
3. A factorable trinomial with a leading coefficient that is not 1, and a “*c*” term (the last term in the trinomial) that has six different trivial factors (for example, 12 has six factors: 1,2,3,4,6,12)
4. A trinomial that can’t be factored using the techniques we’ve learned so far
5. A trinomial where a greatest common factor must first be factored out of each term before the trinomial can be factored
6. A polynomial that can be factored by grouping
7. A polynomial that can be factored by grouping only after the terms have had their order rearranged
8. The polynomial $x^2 - 2x - xy + 2y$ can be factored by grouping if we factor an x out of the first two terms and a $-y$ out of the last two terms, like this: $x(x - 2) + -y(x - 2)$. Come up with a polynomial that can be factored by grouping only if we factor a negative sign out of one part of the polynomial during the first step, like we did above with $-y$.

Rotate roles for each number: **For each problem, the *Prover* should create the polynomial to be factored, the *Explainer* should use the appropriate method to factor the expression completely, and the *Checker* should check the work.**

PART II: Simplifying Rational Expressions**To do this part of project, all we need to know is:**

- How to factor polynomials, and;
- The basic rules and properties for reducing, adding, subtracting, multiplying, and dividing fractions.

All we need to do is to extend what we know about polynomials and about fractions that contain only numbers to fractions that contain polynomials!

Our goal on this part of the assignment: To understand how to simplify rational expressions.

We will use the same rules we use to reduce fractions to reduce rational expressions: factor out the largest common factor of both the numerator and the denominator of the rational expression!

- I. Here is an example of how a *Prover* and an *Explainer* would reduce the rational expression $\frac{2x^2-5x+3}{-x^2+5x-6}$:

Step 1a: Factor the numerator by the *ac*-method:

$$\begin{aligned} 2x^2 - 5x - 3 & \quad \text{By the ac-method, we first find factors that equal } 2 \cdot -3 = -6, \text{ and choose the one whose sum is} \\ & \quad -5. \text{ The pair of factors we find which satisfies these criteria is } -6 \text{ and } 1. \\ = 2x^2 - 6x + 1x - 3 & \quad \text{The middle term is then broken into } -5x = -6x + 1x. \\ = (2x + 1)(x - 3) & \quad \text{We then factor by grouping to obtain the expression to the left.} \end{aligned}$$

Step 1b: Factor the denominator by first factoring out -1 :

$$\begin{aligned} -x^2 + 5x - 6 & \quad \text{Factor } -1 \text{ out first to have leading coefficient one. Then we find integers that multiplied together} \\ = -(x^2 - 5x + 6) & \quad \text{give 6 but add together to get } -5. \text{ The integers that satisfy these two conditions are } -2 \text{ and } -3. \\ = -(x - 2)(x - 3) & \quad \text{Therefore we obtain the expression to the left.} \end{aligned}$$

Step 2: Write the rational expression with numerator and denominator as products:

$$\frac{(2x-1)(x-3)}{-(x-2)(x-3)} \quad \text{We note that here } x \neq 2, 3.$$

Step 3: Cancel common terms in numerator and denominator using the equality property of rational expressions $\left(\frac{ac}{bc} = \frac{a}{b}\right)$:

$$\frac{(2x+1)\overset{1}{\cancel{(x-3)}}}{-(x-2)\overset{1}{\cancel{(x-3)}}} = \frac{(2x+1) \cdot 1}{-(x-2) \cdot 1} = \frac{2x+1}{-(x-2)} \quad \text{Cancel out } \frac{x-3}{x-3} = 1, \text{ leaving all other terms in their place.}$$

Step 4: Simplify if possible:

$$= \frac{2x+1}{2-x} \quad \text{Distribute the factor } (-1) \text{ in the denominator and reorder to have the positive term first.}$$

Now you try! Use example I above as a model to show how to reduce the following rational expressions, rotating through the *Prover*, *Explainer* and *Checker* roles (follow the exact roles given in the *Project Instruction Sheet*):

11. $\frac{3x^3-16x^2-12x+64}{3x^2-22x+32}$.

12. $\frac{x^2-4}{4-x^2}$

13. $\frac{2x^2+18x-72}{3x^2+30x-72}$

14. $\frac{9x^3+9x^2-4x-4}{2-x-3x^2}$