Lab #5: the Derivative as a Function
Calculus 1, Professor Wladis

Please type all your answers
On a separate sheet, please draw by hand f(x) and f '(x) for all the questions

1. Let's get used to using this mathlet:

You can **make your own function:** lets graph $f(x) = x^2 + 6x$.

In the upper right area, click 'demo examples' and select 'user defined function'. A box appears on the bottom. Type in a function, and click 'graph'.

Always use * for multiplication: type "6*x" not "6x".

Do not type "y=" or "f(x)=".

So for this example, type *x^2+6*x* (then click 'graph').

If you need help with how to type in a function, click 'syntax'.

The viewing window {xmin, xmax, ymin, ymax} appears in boxes around the edge of the window.

You can **change the viewing window**: enter new values, then click 'graph'.

For example, try: xmin=-15 xmax=15 ymin=-15 ymax=15.

To find out the **coordinates of a point**, put your mouse over that point. The values appear in the lower right corner of the graph. This means you can get the approximate coordinates of any point on the graph.

a) Using the grapher, when x=-1.26, what is the approximate value of f(x)?

The program will show you the **tangent line at any point** as well as its slope.

Drag the slider at the bottom. Above it is a little tangent line at the given point.

Next to the short tangent line is the approximate **value of the slope**.

The program then plots each value of the slope (derivative). That graph is green: it graphs the value of the slope on the y-axis for each x-value.

b) When x = -1.26, what is the approximate slope of f(x) ?

You can try to **draw the graph of the derivative yourself**:

With your mouse in the window, click and hold, then drag.

Tip: Don't drag the mouse too fast.

Because we will be talking about several functions at once, we need to give them different names (they can't all be "y=" that would be confusing): We will call a function *f(x)*. **Its derivative is called *f '(x)*** "f-prime of x".

2. Small technical problems

Graph *f(x) = round(x)* ...this rounds x to the nearest integer.

a) What should the graph look like?

b) How does the computer draw the graph?

This happens because this grapher "connects the dots" (as we saw on lab#1).

c) What should the slope be at these "problem spots"? What does the computer say the slope is?

Click 'reset'.

3. Graph $f(x) = 4$ ... type *4* then click 'graph'. a) What is the slope at x=-6?
b) What is the slope at x=-4?
c) What is the slope for any x-value?
Try to finish drawing the graph of *f '(x)* with the mouse.
Drag the slider along the graph. (Did you get it right?)
d) What is the value of *f '(x)* at each x-value?
e) Write the formula for the derivative function.

4. Graph $f(x) = 2x + 3$ ... type *2*x+3* then click 'graph'
a) What is the slope at x=1?
b) What is the slope at x=2?
c) What is the slope for any x-value?
Try to finish drawing the graph of *f '(x)* with the mouse.
Drag the slider along the graph. (Did you get it right?)
d) what is the value of *f '(x)* at each x-value?
e) Write the formula for the derivative function.

5. Graph $f(x) = arctan(x)$ ... type *atan(x).*
Change the viewing window to xmin=-4 xmax=4 ymin=-4 ymax=4, click 'graph'. a) Describe the graph in words. What is always true about the graph of f(x) ?
Try to draw *f '(x)* with the mouse.
Drag the slider to graph *f '(x)* and see if you were right.
b) What is always true about the graph of *f '(x)* ?
Click 'reset'

6. Graph $f(x) = -1.008 + 1.04x + 0.047x^2 - 0.081x^3 + 0.001x^4 + 0.001x^5$.
a) Describe the graph in one sentence.
b) Drag the slider. Describe the graph of the derivative in one sentence.
c) How are the two graphs the same? How are they different?
d) *f(x)* is described by what kind of formula?
e) The formula for *f '(x)*, how do you think it is the same as the formula for f(x)? How is it different?
Trying to figure out the slope at a point by looking at the original graph is hard, and you have to estimate. There are some points on a graph, however, where it's easy to say exactly what the slope is:
f) Which points are those (describe in words or give x-values)? What is the value of the slope at these points?

Note: answer questions about "for which x-values" with approximate answers.

7. Graph $f(x) = 1.92 + 1.52x - 0.46x^2 - 0.05x^3 + 0.01x^4$.
   a) For which x-values is the function increasing? (e.g. "from a to b")
   b) For which x-values is the function decreasing?
   c) What does this tell you about the graph of *f '(x)* ?

Try to draw *f '(x)* with the mouse.
Drag the slider to see if you are right.

8. Graph $f(x) = -2.37341 - 1.2515x + 0.67008x^2 + 0.0396x^3 - 0.012x^4$.
   a) For which x-values is *f(x)* increasing?
   b) What does that tell you about the derivative function *f '(x)* ?
   c) For which x-values is *f(x)* decreasing?
   d) What does that tell you about the derivative function *f '(x)*?
   e) For which x-values is *f(x)* increasing faster and faster?
   f) What does that tell you about the derivative function *f '(x)*?
   g) For which x-values is *f(x)* increasing slower and slower?
   h) What does that tell you about the derivative function *f '(x)*?
   i) For which x-values is *f(x)* decreasing faster and faster?
   j) What does that tell you about the derivative function *f '(x)*?
   k) For which x-values is *f(x)* decreasing slower and slower?
   l) What does that tell you about the derivative function *f '(x)*?

9. Graph $f(x) = 2.1172 - 2.1652x + 0.008x^2 + 0.04x^3$.
   a) For which x-values will *f '(x)* be 0?
   b) For which x-values will *f '(x)* be positive ?
   c) For which x-values will *f '(x)* be negative ?
   d) For which x-values will *f '(x)* be increasing ?
   e) For which x-values will *f '(x)* be decreasing ?

Draw *f '(x)* with the mouse.
Drag the slider to see if you are right.

10. Graph $f(x) = |x - 2| + 2x$ by typing *f(x) = abs(x-2)+2*x.*
   Try to draw *f '(x)* with the mouse.
   Drag the slider to see if you are right.
   a) At what x-value does something unusual happen? Describe in one sentence what the grapher draws for *f '(x)* then describe in one sentence what it *should* draw.

Here are some more functions to graph, then try to draw *f '(x)*:
11. $f(x) = x^3$
12. $f(x) = |2 \sin x| - 1$, *or f(x)=abs(2*sin(x))-1*
13. $f(x) = \sin x + x$, or *f(x)=sin(x)+x*

So far we have graphed *f '(x)* and made general descriptions in words. Some graphs help us be precise.

Click 'reset'.

14. Graph $f(x) = x^2$.

Drag the slider all the way across.

a) Describe in words the graph of *f '(x).*

b) Can you give a formula for *f '(x)*?

15. Graph *f(x) = sin(x).*

Try to draw the graph of *f '(x)* with the mouse.

Drag the slider all the way across to see if you were right.

a) Can you guess the formula for *f '(x)* ?

16. Graph *f(x) = cos(x).*

Try to draw the graph of *f '(x)* with the mouse.

Drag the slider all the way across to see if you were right.

a) Can you guess the formula for *f '(x)* ?

Click 'reset'.

17. Graph $f(x) = 2^x$.

Drag the slider.

a) Make a comparison between the graph of the function and the graph of the derivative.

18. Graph $f(x) = 3^x$.

Drag the slider.

a) Make a comparison between the graph of the function and the graph of the derivative.

19. Graph $f(x) = 2.7^x$.

Drag the slider.

a) Make a comparison between the graph of the function and the graph of the derivative.

Change the viewing window to xmin=-.1 xmax=.1 ymin=.9 ymax=1.1, and click 'graph'.

b) Now describe in words what you see. Click 'reset'.

Recall that the number *e* is approximately 2.718281828...

20. Graph $f(x) = e^x$.

Drag the slider.

a) Make a comparison between the graph of the function and the graph of the derivative. Change the viewing window to xmin=-.1 xmax=.1 ymin=.9 ymax=1.1, and click 'graph'.

Drag the slider.

b) Make a comparison between the graph of the function and the graph of the derivative.

c) Make a conclusion about the function and its derivative using “f(x)” notation.

For the following problems, you may want to compare the graphs of different functions. Open up a second window by right-clicking on the "technology tool" link on my website (with a mac, use apple-click).
These problems below may lead to some interesting conclusions:

21. Open two windows so you can compare two functions (turn in graphs for f & f ' and g & g')

Graph $f(x) = 0.08x^3 - 2x$.

Try to draw *f '(x)* with the mouse. Drag the slider to see if you are right.

Graph $g(x) = .08x^3 - 2x + 3$.

Try to draw *g'(x)* with the mouse. Drag the slider to see if you are right.

a) What is the difference between *f(x)* and *g(x)* ?
b) What is the difference between *f '(x)* and *g'(x)* ?
c) Can you explain why this makes sense?

22. Open three windows so you can compare three functions (turn in f&f ' g&g' h&h')

Graph $f(x)=x^2$.

Try to draw *f '(x)* with the mouse. Drag the slider to see if you are right.

a) Try to guess the equation for *f '(x).*

Graph *g(x)=3x.*

Try to draw *g'(x)* with the mouse. Drag the slider to see if you are right.

b) Try to guess the equation for *g'(x).*

Graph $h(x)=x^2+3x$.

Try to draw *h'(x)* with the mouse. Drag the slider to see if you are right.

c) Try to guess the equation for *h'(x).*
d) How do *f(x)* and *g(x)* relate to *h(x)*?
e) How do *f '(x)* and *g'(x)* relate to *h'(x)*?

23. *[extra credit]* Open three windows so you can compare three functions (turn in f&f ' g&g' h&h')

a) Graph $f(x)=1.5x$.
   Try to draw $f'(x)$ with the mouse. Drag the slider to see if you are right.

b) Try to guess the equation for $f'(x)$.

c) Graph $g(x)=2x$.
   Try to draw $g'(x)$ with the mouse. Drag the slider to see if you are right.

d) Try to guess the equation for $g'(x)$.

e) Graph $h(x)=3x^2$.
   Try to draw $h'(x)$ with the mouse. Drag the slider to see if you are right.

f) Try to guess the equation for $h'(x)$.

g) How do $f(x)$ and $g(x)$ relate to $h(x)$?

h) How do $f'(x)$ and $g'(x)$ NOT relate to $h'(x)$? (or, what is something you might guess is true but in this example is clearly not true)