

**Lab #3: Limits**  
**Calculus I, Prof. Wladis**  
*Please type all your answers*

**Part I of this Lab:** Do all the **a** and **b** parts below. Leave room for the **c** parts to be added next week. Submit this file for the deadline for Part I.

**Part II of this Lab:** Take the file you submitted for Part I of this lab, and **add in** your answers to all the **c** parts below. Submit the whole file with all of your a, b, and c answers for the deadline for Part II.

Let's get to work:

1. Graph  $y = x^{-2}$ :

Type in  $y=x^{(-2)}$  and hit the "graph" button.

In the box labeled "c," you can type in x-values to see what the corresponding value of y is for a particular value of x.

a. Look at the graph: What do you think happens as x approaches zero from the left? When x approaches zero from the right? Describe this in a few sentences.

b. Make a table of y values for each of the following x values (by typing these values in the "c" box): 1, 0.1, 0.01, 0.001, 0.0001.

Now make another table of values for each of the following x values: -2, -0.2, -0.02, -0.002, -0.0002.

Describe in words what you think happens to y as x gets closer to zero from the right, and what happens when x gets closer to zero from the left, based on what you see in these tables of values.

c. Evaluate the limits algebraically, as we have done in class:

$$\text{Find } \lim_{x \rightarrow 0^-} x^{-2} = ?$$

$$\text{Find } \lim_{x \rightarrow 0^+} x^{-2} = ?$$

2. Graph  $y = x^{-1}$ :

Type in  $y=x^{(-1)}$  and hit the "graph" button.

a. Look at the graph: What do you think happens as x approaches zero from the left? When x approaches zero from the right? Describe this in a few sentences.

b. Make a table of y values for each of the following x values (by typing these values in the "c" box): 3, 0.3, 0.03, 0.003, 0.0003.

Now make another table of values for each of the following x values: -2, -0.2, -0.02, -0.002, -0.0002.

Describe in words what you think happens to y as x gets closer to zero from the right, and what happens when x gets closer to zero from the left, based on what you see in these tables of values.

c. Evaluate the limits algebraically, as we have done in class:

$$\text{Find } \lim_{x \rightarrow 0^-} x^{-1} = ?$$

$$\text{Find } \lim_{x \rightarrow 0^+} x^{-1} = ?$$

3. Graph  $y = \frac{2x}{\sqrt{x^2-1}}$ :

Type in  $y=2x/(x^2-1)^{0.5}$  and hit the "graph" button.

a. Look at the graph: What do you think happens as x increases without bound? When x decreases without bound? Describe this in a few sentences.

- b. Make a table of y values for each of the following x values (by typing these values in the “c” box):  
1, 10, 100, 1000, 1,000,000.

Now make another table of values for each of the following x values: -5, -125, -525, -3125, -78,125.

Describe in words what you think happens to y as x increases without bound, and what happens when x decreases without bound, based on what you see in these tables of values.

- c. Evaluate the limits algebraically, as we have done in class:

$$\text{Find } \lim_{x \rightarrow -\infty} \frac{2x}{\sqrt{x^2-1}} = ?$$

$$\text{Find } \lim_{x \rightarrow \infty} \frac{2x}{\sqrt{x^2-1}} = ?$$

The following questions cannot be evaluated algebraically yet, because we haven't learned how to find the limits of these kinds of functions yet. However, we can use both the visual graph and sample values to make educated guesses about the limits of these functions.

4. Graph  $y = \frac{e^{2x}-1}{x}$  :

Type in  $y=(e^{2x}-1)/x$  and hit the “graph” button.

- a. Look at the graph, and try zooming in on the graph where  $x=0$ : What do you think happens as x approaches zero from the left and the right? Describe this in a few sentences.  
b. Notice that if you try to look up the value of y when  $x=0$ , the mathlet doesn't give you an answer: this is because this function is not defined when  $x=0$ .

Make a table of y values for each of the following x values (by typing these values in the “c” box):  
1, 0.2, 0.03, 0.004, 0.00005.

Now make another table of values for each of the following x values: -1, -0.05, -0.008, -0.0000079, -0.00000001.

Describe in words what you think happens to y as x gets closer to zero from the right and from the left, based on what you see in these tables of values.

- c. Based on your answers to a) and b), make an educated guess about the following limit:

$$\text{Find } \lim_{x \rightarrow 0} \frac{e^{2x}-1}{x} = ?$$

5. Graph  $y = \frac{\ln x}{x}$ :

Type in  $y=\ln(x)/x$  and hit the “graph” button.

- a. Look at the graph: What do you think happens as x increases without bound? Describe this in a few sentences.  
b. Make a table of y values for each of the following x values (by typing these values in the “c” box):  
2, 120, 500, 8001, 325,000.  
Describe in words what you think happens to y as x increases without bound, based on what you see in these tables of values.

- c. Based on your answers to a) and b), make an educated guess about the following limit:

$$\text{Find } \lim_{x \rightarrow \infty} \frac{\ln x}{x} = ?$$

6. Graph  $y = x^x$  :

Type in  $y=x^x$  and hit the “graph” button.

- a. Look at the graph, and try zooming in on the graph where  $x=0$ : What do you think happens as x approaches zero from the right? Describe this in a few sentences.

b. Notice that if you try to look up the value of  $y$  when  $x=0$ , the mathlet doesn't give you an answer: this is because this function is not defined when  $x=0$ .

Make a table of  $y$  values for each of the following  $x$  values (by typing these values in the "c" box): 2, 0.4, 0.06, 0.008, 0.0001.

Describe in words what you think happens to  $y$  as  $x$  gets closer to zero from the right, based on what you see in these tables of values.

c. Based on your answers to a) and b), make an educated guess about the following limit:

$$\text{Find } \lim_{x \rightarrow 0^+} x^x = ?$$

7. Graph  $y = \frac{e^x}{x}$ :

Type in  $y=e^x/x$  and hit the "graph" button.

a. Look at the graph: What do you think happens as  $x$  increases without bound? Describe this in a few sentences.

b. Make a table of  $y$  values for each of the following  $x$  values (by typing these values in the "c" box): 1, 50, 800, 9999, 3,600,000.

Describe in words what you think happens to  $y$  as  $x$  increases without bound, based on what you see in these tables of values.

c. Based on your answers to a) and b), make an educated guess about the following limit:

$$\text{Find } \lim_{x \rightarrow \infty} \frac{e^x}{x} = ?$$

8. Graph  $y = \sin \frac{1}{x}$ :

Type in  $y=\sin(1/x)$  and hit the "graph" button.

Then set the Bounds to  $x_{\min}=-1$ ,  $x_{\max}=1$ ,  $y_{\min}=-1$ ,  $y_{\max}=1$ .

a. Look at the graph: what do you think happens as  $x$  approaches zero?

b. Change the bounds to  $x_{\min}=-0.1$ ,  $x_{\max}=0.1$ ,  $y_{\min}=-1$ ,  $y_{\max}=1$ , and look at the resulting graph. Repeat this with the bounds  $x_{\min}=-0.01$ ,  $x_{\max}=0.01$ ,  $y_{\min}=-1$ ,  $y_{\max}=1$ ,  $x_{\min}=-0.001$ ,  $x_{\max}=0.001$ ,  $y_{\min}=-1$ ,  $y_{\max}=1$ , and  $x_{\min}=-0.0001$ ,  $x_{\max}=0.0001$ ,  $y_{\min}=-1$ ,  $y_{\max}=1$ . Describe in words what you think the value of  $y$  does as  $x$  approaches zero.

c. What do you think happens to  $f(x)$  as  $x \rightarrow 0$ ? Do you think that the limit exists? Why or why not? (If it does exist, explain what the limit is.)

9. Please write one paragraph about what you learned during this lab, and which activities helped you learn it.

10. Was anything in this lab confusing? Is there anything you still don't understand? What would you change?