Lab #2: Formal Definition of Limits
Calculus I, Prof. Wladis

1. Graph $y = -3^x$:

In the box labeled "c," you can type in x-values to see what the corresponding value of y is for a particular value of x.

- **a.** Find the limit L as $x \to 2$ algebraically, or show that it does not exist.
- **b.** Let $\epsilon = 0.01$. If the two-sided limit does NOT exist, use the formal definition of the limit to explain **why** it does not exist. If the two-sided limit does NOT exist, but each of the one-sided limits DO exist, then just do parts c., e., and g. below for each one-sided limit. If neither one-sided limits also do NOT exist, then skip the remaining parts of this question.
- **c.** If the limit **does** exist, use the graph to estimate the value of δ_1, the δ-value needed to satisfy the formal limit definition for the left-sided limit.
- **d.** Use the graph to estimate the value of δ_2, the δ-value needed to satisfy the formal limit definition for the right-sided limit.
- **e.** If δ_1 and δ_2 are not the same, explain which of these values you will need to use as your δ-value in order to satisfy the formal definition of the two-sided limit.
- **f.** Now find δ_1 and δ_2 exactly by calculating them algebraically.
- **g.** Give the exact value of δ needed to satisfy the formal limit definition for the two-sided limit, rounded to three decimal places if it is an irrational number.
- **h.** Give two other values of δ which would also satisfy the formal limit definition for this function and this value of ϵ.

2. Graph $y = \begin{cases} -3^x & x < 1 \\ 2x - 1 & x \geq 1 \end{cases}$:

To graph this, you will need to type in each equation separately (in the y_1 and y_2 fields), and then be careful to think about for which values of x, and therefore for which parts of the graph, each equation actually applies.

In the box labeled "c," you can type in x-values to see what the corresponding value of y is for a particular value of x.

- **a.** Find the limit L as $x \to 1$ algebraically, or show that it does not exist.
- **b.** Let $\epsilon = 0.01$. If the two-sided limit does NOT exist, use the formal definition of the limit to explain **why** it does not exist. If the two-sided limit does NOT exist, but each of the one-sided limits DO exist, then just do parts c., e., and g. below for each one-sided limit. If neither one-sided limits also do NOT exist, then skip the remaining parts of this question.
- **c.** If the limit **does** exist, use the graph to estimate the value of δ_1, the δ-value needed to satisfy the formal limit definition for the left-sided limit.
- **d.** Use the graph to estimate the value of δ_2, the δ-value needed to satisfy the formal limit definition for the right-sided limit.
- **e.** If δ_1 and δ_2 are not the same, explain which of these values you will need to use as your δ-value in order to satisfy the formal definition of the two-sided limit.
- **f.** Now find δ_1 and δ_2 exactly by calculating them algebraically.
- **g.** Give the exact value of δ needed to satisfy the formal limit definition for the two-sided limit, rounded to three decimal places if it is an irrational number.

h. Give two other values of δ which would also satisfy the formal limit definition for this function and this value of ϵ.

3. Graph $y = (x-2)^{-2}$:

In the box labeled "c," you can type in x-values to see what the corresponding value of y is for a particular value of x.

a. Find the limit L as $x \to 2$ algebraically, or show that it does not exist.

b. Let $\epsilon = 0.1$. If the two-sided limit does NOT exist, use the formal definition of the limit to explain **why** it does not exist. If the two-sided limit does NOT exist, but each of the one-sided limits DO exist, then just do parts c., e., and g. below for each one-sided limit. If neither one-sided limits also do NOT exist, then skip the remaining parts of this question.

c. If the limit **does** exist, use the graph to estimate the value of δ_1, the δ-value needed to satisfy the formal limit definition for the left-sided limit.

d. Use the graph to estimate the value of δ_2, the δ-value needed to satisfy the formal limit definition for the right-sided limit.

e. If δ_1 and δ_2 are not the same, explain which of these values you will need to use as your δ-value in order to satisfy the formal definition of the two-sided limit.

f. Now find δ_1 and δ_2 exactly by calculating them algebraically.

g. Give the exact value of δ needed to satisfy the formal limit definition for the two-sided limit, rounded to three decimal places if it is an irrational number.

h. Give two other values of δ which would also satisfy the formal limit definition for this function and this value of ϵ.

4. Graph $y = (2x+4)^{-1}$:

In the box labeled "c," you can type in x-values to see what the corresponding value of y is for a particular value of x.

a. Find the limit L as $x \to -2$ algebraically, or show that it does not exist.

b. Let $M = 50$. If the two-sided limit does NOT exist, use the formal definition of the limit to explain **why** it does not exist. If the two-sided limit does NOT exist, but each of the one-sided limits DO exist, then just do parts c., e., and g. below for each one-sided limit. If neither one-sided limits also do NOT exist, then skip the remaining parts of this question.

c. If the limit **does** exist, use the graph to estimate the value of δ_1, the δ-value needed to satisfy the formal limit definition for the left-sided limit.

d. Use the graph to estimate the value of δ_2, the δ-value needed to satisfy the formal limit definition for the right-sided limit.

e. If δ_1 and δ_2 are not the same, explain which of these values you will need to use as your δ-value in order to satisfy the formal definition of the two-sided limit.

f. Now find δ_1 and δ_2 exactly by calculating them algebraically.

g. Give the exact value of δ needed to satisfy the formal limit definition for the two-sided limit, rounded to three decimal places if it is an irrational number.

h. Give two other values of δ which would also satisfy the formal limit definition for this function and this value of M.

5. Graph $y = \frac{-3x}{\sqrt{x^2-4}}$:

In the box labeled "c," you can type in x-values to see what the corresponding value of y is for a particular value of x.

- **a.** Find the limit L as $x \to +\infty$ algebraically, or show that it does not exist.
- **b.** Let $\epsilon = 0.1$. If the limit does NOT exist, use the formal definition of the limit to explain **why** it does not exist.
- **c.** If the limit **does** exist, use the graph to estimate the value of M.
- **d.** Give two other values of M which would also satisfy the formal limit definition for this function and this value of ϵ.

6. Graph $y = \frac{-x^3-1}{x^2+4}$:

In the box labeled "c," you can type in x-values to see what the corresponding value of y is for a particular value of x.

- **a.** Find the limit L as $x \to +\infty$ algebraically, or show that it does not exist.
- **b.** Let $M_y = 100$. If the limit does NOT exist, use the formal definition of the limit to explain **why** it does not exist.
- **c.** If the limit **does** exist, use the graph to estimate the value of M_x.
- **d.** Give two other values of M_x which would also satisfy the formal limit definition for this function and this value of M_y.

7. Graph $y = \sin\frac{1}{x}$:

In the box labeled "c," you can type in x-values to see what the corresponding value of y is for a particular value of x.

- **a.** Find the limit L as $x \to 0$ algebraically, or show that it does not exist.
- **b.** Let $\epsilon = 0.2$. If the two-sided limit does NOT exist, use the formal definition of the limit to explain **why** it does not exist. If the two-sided limit does NOT exist, but each of the one-sided limits DO exist, then just do parts c., e., and g. below for each one-sided limit. If neither one-sided limits also do NOT exist, then skip the remaining parts of this question.
- **c.** If the limit **does** exist, use the graph to estimate the value of δ_1, the δ-value needed to satisfy the formal limit definition for the left-sided limit.
- **d.** Use the graph to estimate the value of δ_2, the δ-value needed to satisfy the formal limit definition for the right-sided limit.
- **e.** If δ_1 and δ_2 are not the same, explain which of these values you will need to use as your δ-value in order to satisfy the formal definition of the two-sided limit.
- **f.** Now find δ_1 and δ_2 exactly by calculating them algebraically.
- **g.** Give the exact value of δ needed to satisfy the formal limit definition for the two-sided limit, rounded to three decimal places if it is an irrational number.
- **h.** Give two other values of δ which would also satisfy the formal limit definition for this function and this value of ϵ.