Lab #2: Formal Definition of Limits
Calculus I, Prof. Wladis

For each of the following functions, answer each of the questions below:

a. Find the left-sided limit as $x \to c$ either graphically or algebraically. If the limit increases or decreases without bound, write inf or -inf, respectively. If it does not exist and cannot be represented by $+\infty$ or $-\infty$, enter DNE.
   i. If the limit does not exist, explain why:
      1. The function is not defined around c
      2. The function's behavior around c does not approach one set value
   ii. If the left-sided limit exists, graph the function to estimate the value of δ_1, the δ-value needed to satisfy the formal limit definition for the left-sided limit for the given ϵ or M_y, rounded to the nearest thousandths. (Tip: it can help to see where $x = L{-}\in$ if you graph the line $x = L{-}\in$ at the same time as graphing y, and then just look where the two lines intersect, in order to find the corresponding y-value at $x = L{-}\in$.) If the limit does not exist, enter NA.
   iii. If the left-sided limit increases or decreases without bound, graph the function to estimate the value of M, the M-value needed to satisfy the formal limit definition for the left-sided limit for the given $\in$, rounded to the nearest thousandths. (Tip: it can help to see where $x = L{-}\in$ if you graph the line $x = L{-}\in$ at the same time as graphing y, and then just look where the two lines intersect, in order to find the corresponding y-value at $x = L{-}\in$.) If the limit does not exist, enter NA.

b. Find the right-sided limit either graphically or algebraically. If the limit increases or decreases without bound, write inf or -inf, respectively. If it does not exist and cannot be represented by $+\infty$ or $-\infty$, enter DNE.
   i. If the limit does not exist, explain why:
      1. The function is not defined around c
      2. The function's behavior around c does not approach one set value
   ii. If the right-sided limit exists, graph the function to estimate the value of δ_1, the δ-value needed to satisfy the formal limit definition for the right-sided limit for the given ϵ or M_y, rounded to the nearest thousandths. (Tip: it can help to see where $x = L{-}\in$ if you graph the line $x = L{-}\in$ at the same time as graphing y, and then just look where the two lines intersect, in order to find the corresponding y-value at $x = L{-}\in$.) If the limit does not exist, enter NA.
   iii. If the right-sided limit increases or decreases without bound, graph the function to estimate the value of M_1, the M-value needed to satisfy the formal limit definition for the right-sided limit for the given $\in$, rounded to the nearest thousandths. (Tip: it can help to see where $x = L{-}\in$ if you graph the line $x = L{-}\in$ at the same time as graphing y, and then just look where the two lines intersect, in order to find the corresponding y-value at $x = L{-}\in$.) If the limit does not exist, enter NA.

c. Find the two-sided limit. If the limit increases or decreases without bound, write inf or -inf, respectively. If it does not exist and cannot be represented by $+\infty$ or $-\infty$, enter DNE.

i. If the limit does not exist, explain why:

1. The function is not defined around c
2. The function's behavior around c does not approach one set value
3. The left and right limits are not equal

ii. If the two-sided limit exists, give the maximum δ or M that satisfies the formal limit definition for both the left and the right sided limits. If the two-sided limit does not exist, enter NA.

iii. If the two-sided limit exists, give two further values for δ or M that also satisfy the formal limit definition for the two-sided limit. If the two-sided limit does not exist, enter NA.

1. $f(x) = x^2 + 1, c = 0, \ \in = 0.01, M_y = 10$

2. $f(x) = -3^x, c = 2, \ \in = 0.01, M_y = 10$

3. $f(x) = \sqrt{x+2}, c = -6, \ \in = 0.01, M_y = 10$

4. $f(x) = \begin{cases} -3^x & x < 1 \\ 2x - 1 & x \geq 1 \end{cases}, c = 1, \ \in = 0.01, M_y = 10$

To graph this, depending on what software you are using, you may need to type in each equation separately (in the y_1 and y_2 fields), and then be careful to think about for which values of x, and therefore for which parts of the graph, each equation actually applies. If you are using Desmos, you can type this in as follows: y = {condition: value, condition: value, etc.}. So for this function, for example, you would type: y = {x<1: -3^x, x>=1: 2x-1}

5. $f(x) = \begin{cases} (x-2)^{-1} & x < 2 \\ (x-2)^{-2} & x \geq 2 \end{cases}, c = 2, \ \in = 0.01, M_y = 100$

6. $f(x) = (x-2)^{-2}, c = 2, \ \in = 0.1, M_y = 100$

7. $f(x) = (2x+4)^{-1}, c = 2, \ \in = 0.01, \ M_y = 50$

8. $f(x) = \frac{-3x}{\sqrt{x^2-4}}, c = +\infty, \in = 0.1, M_y = 50$

9. $f(x) = \frac{-x^3-1}{x^2+4}, c = +\infty, \ \in = 0.1, \ M_y = 100$

10. $f(x) = \sqrt{x+10}, c = -\infty, \ \in = 0.1, M_y = 100$

11. $f(x) = \frac{50\sin(2x)}{x}, c = -\infty, \ \in = 0.5, M_y = 10$

12. $f(x) = \sin x, c = -\infty, \ \in = 0.1, M_y = 10$