Simplifying Rational Expressions

What is a <u>rational</u> expression?

<u>Defn</u>: A <u>rational expression</u> is an expression that can be written in the form

$$\frac{P}{Q}$$ for some polynomials P & Q ($Q \neq 0$)

Why are we interested in rational expressions?

Rates:
- speed/velocity: mph, ft/sec
- miles per gallon
- dollars per month

What do we want to be able to do with rational expressions?

- simplify them (this lecture)
- perform operations on them ($+, -, \cdot, \div$)
- solve equations that include them

What does it mean to <u>simplify</u> a <u>rational</u> <u>expression</u>?

Rational expressions are just fractions, so we simplify them in the same way: by cancelling out common factors that appear in both the numerator and the denominator.

<u>Example</u>:

$$\frac{12}{30} = \frac{\overset{1}{\cancel{6}} \cdot 2}{\underset{1}{\cancel{6}} \cdot 5} = \boxed{\frac{2}{5}} \leftarrow$$

Examples

1) $\dfrac{x^2-16}{4-x}$ } Simplify.

$$= \frac{(x+4)(x-4)}{4-x} = \frac{(x+4)\overset{1}{\cancel{(x-4)}}}{-\underset{1}{\cancel{(x-4)}}} = \frac{x+4}{-1} = -(x+4)$$

$$= \boxed{4-x} \quad \text{or} \quad \boxed{-x+4}$$

Note!

In the original expression, $x \neq 4$ (because then we would get zero in the denominator), but in our simplified expression we can have $x = 0$.

Sometimes, when we simplify an expression, it changes what values one allowed to be put in for the variable

2) Simplify:

$$\frac{2x^2+5x-3}{2x^2+3x-2}=\frac{(\overset{1}{\cancel{2x-1}})(x+3)}{(x+2)(\underset{1}{\cancel{2x-1}})}=\boxed{\frac{x+3}{x+2}}$$

Note! $2x-1\neq0$ in the original expression.

3) Simplify:

$$\frac{3x^2+3x-60}{9x^2+72x+135}=\frac{\overset{1}{\cancel{3}}(x-4)(\cancel{x+5})^{1}}{\underset{}{\overset{3}{\cancel{9}}}(\underset{1}{\cancel{x+5}})(x+3)}=\boxed{\frac{x-4}{3(x+3)}}$$

or $\boxed{\dfrac{x-4}{3x+9}}$

Note! $x\neq-5$ in the original expression.

4) Simplify:

$$\frac{x^2-3x+xy-3y}{x^2-xy-3x+3y}=\frac{(x+y)(\cancel{x-3})^{1}}{(\cancel{x-3})(x-y)}=\boxed{\frac{x+y}{x+y}}$$

Note! $x\neq3$ in the original expression.

5) Simplify:

$$\frac{x-2}{x^2-4}=\frac{\cancel{x-2}^{1}}{(x+2)(\underset{1}{\cancel{x-2}})}=\boxed{\frac{1}{x+2}}\leftarrow$$

Note! $x\neq2$ in the original expression

6) Simplify:

$$\frac{y-x}{x-y}=\frac{-\overset{1}{(\cancel{x-y})}}{\underset{1}{\cancel{x-y}}}=\frac{-1}{1}=\boxed{-1}$$

Note! $x\neq y$ in the original expression.