Dividing Polynomials

Fractions are just a representation of division.

So: A rational expression $\frac{P}{Q}$ is really just division of polynomials $P \div Q$.

With numbers, if we have a fraction that is improper, then we can rewrite that fraction as a mixed number.

We can do something similar with rational expressions:

Example: $\frac{12}{5} \rightarrow$ simplified

If we prefer, we can write $12 \div 5$ and perform long division to turn this into a mixed number.

$$\begin{array}{r} 2\tfrac{2}{5} \\ 5\,\overline{)\,12} \\ \underline{-10} \\ 2 \end{array}$$

2 ← remainder

$12 \div 5 = 2\frac{2}{5}$ → mixed number

improper fraction → $\boxed{\frac{12}{5} = 2\frac{2}{5}}$

both simplified

Examples with polynomials in rational expressions!

Divide:

1) $$\frac{3x^2-5x+2}{x-3}$$

$$= \boxed{3x+4+\frac{14}{x-3}}$$

Long division for polynomials.

$$
\begin{array}{r l}
 & 3x+4+\frac{14}{x-3} \\
x-3 \,\big) & 3x^2-5x+2 \\
 & +(-3x^2+9x) \\
\hline
 & 0+4x+2 \\
 & +(-4x+12) \\
\hline
 & 0+14 \leftarrow \text{remainder}
\end{array}
$$

Why does this work:

$$3x^2-5x+2 = \overbrace{3x(x-3)}^{3x^2-9x} + (4x+2)$$

$$= \underbrace{3x(x-3)+\overbrace{4(x-3)}^{4x-12}}+\underline{\underline{14}}$$

$$\frac{3x^2-5x+2}{x-3} = \frac{3x(x-3)+4(x-3)+14}{x-3}$$

$$= \frac{3x\cancel{(x-3)}}{\cancel{x-3}} + \frac{4\cancel{(x-3)}}{\cancel{x-3}} + \frac{14}{x-3}$$

$$= \boxed{3x+4+\frac{14}{x-3}}$$

2) Divide: $\dfrac{8x^3-27}{2x-1}$

$$
\begin{array}{r|l}
 & 4x^2+2x+1 \\
\hline
2x-1 & 8x^3+0x^2+0x-27 \\
 & +(\cancel{-}8x^3+4x^2) \\
\hline
 & 0+4x^2+0x \\
 & +(\cancel{-}4x^2+2x) \\
\hline
 & 0+2x+-27 \\
 & +(\cancel{-}2x+1) \\
\hline
 & 0-26 \leftarrow \text{remainder}
\end{array}
$$

$$\boxed{4x^2+2x+1+\frac{-26}{2x-1}}$$

3) Divide:

$$\frac{x^4-3x^2+2}{x^2-3}$$

$$\boxed{x^2 + \frac{2}{x^2-3}}$$

$$x^2+0x-3 \,\overline{\big)\, x^4+0x^3-3x^2+0x+2}$$

$$+(-x^4-0x^3+3x^2)$$

$$0+0+0 \quad 0x+2 \rightarrow \text{remainder}$$

4) Divide:

$$\frac{9x^3-6x^2+13x-5}{3x+2}$$

$$\boxed{3x^2-4x+7+\frac{-19}{3x+2}}$$

$$3x+2 \,\overline{\big)\, 9x^3-6x^2+13x-5}$$

$$+(-9x^3+6x^2)$$

$$0+{}^-12x^2+13x$$

$$+(+12x^2+8x)$$

$$0+21x-5$$

$$+(-21x-14)$$

$$0+{}^-19 \rightarrow \text{remainder}$$