Project 7: Identifying Minigoals in order to Rewrite Expressions

Throughout this project, we will use the following properties to simplify expressions. You may want to print this out and keep it next to the project as you work.

$x = 1x \qquad -x = -1x$
$nx = \underbrace{x + \cdots + x}_{n-many\ times}$
$x - x = 0$
$a - b = a + -b$
Multiple instances of addition can be performed in any order. For example: $a + b = b + a \qquad (a + b) + c = a + b + c \qquad a + (b + c) = a + b + c$
Multiple instances of multiplication can be performed in any order. For example: $ab = ba \qquad (ab)c = abc \qquad a(bc) = abc$
$a(b + c) = ab + ac \qquad (a + b)c = ac + bc$ $a(x_1 + \cdots + x_n) = ax_1 + \cdots + ax_n \qquad (x_1 + \cdots + x_n)c = x_1c + \cdots + x_nc$
$x^n = \underbrace{x \cdot \ldots \cdot x}_{n-many\ times}$
$x^{-n} = \frac{1}{x^n} \quad (whenever\ x \neq 0)$
$\frac{x}{1} = x$
$\frac{x}{x} = 1 \quad (whenever\ x \neq 0)$
$\frac{ab}{cd} = \frac{a}{c} \cdot \frac{b}{d} \quad (whenever\ c, d \neq 0) \qquad \frac{x_1 \cdot \ldots \cdot x_n}{y_1 \cdot \ldots \cdot y_n} = \frac{x_1}{y_1} \cdot \ldots \cdot \frac{x_n}{y_n} \quad (whenever\ y_1, \ldots, y_n \neq 0)$
$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad (whenever\ c \neq 0) \qquad \frac{x_1 + \cdots + x_n}{y} = \frac{x_1}{y} + \cdots + \frac{x_n}{y} \quad (whenever\ y \neq 0)$
$\frac{\frac{a}{b}}{c} = \frac{a}{b} \cdot \frac{1}{c} \quad (whenever\ b, c \neq 0) \qquad \frac{a}{\frac{b}{c}} = \frac{a}{1} \cdot \frac{c}{b} \quad (whenever\ b, c \neq 0)$
$\sqrt{x^2} = x \quad (whenever\ x \geq 0)$
$\sqrt{ab} = \sqrt{a}\sqrt{b}$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad (whenver\ b \neq 0)$

A quick bit of background that will help us to simplify radicals:

Factoring numbers under a radical

In order to simplify radicals that have numbers under the radical sign, we will need to be able to factor numbers completely into their simplest parts.

Factoring just means that we rewrite a number as a product of smaller whole numbers. So, for example, 6 could be factored as $6 = 2 \cdot 3$. There is often more than one way to factor a number. For example, 12 can be factored several different ways: $12 = 3 \cdot 4$ or $12 = 2 \cdot 6$. When we factor numbers completely, we break them down into the smallest possible pieces, by continuing to factor every number until we can't go any farther. If we do this with 12 for example, we get: $12 = 2 \cdot 2 \cdot 3$.

Here are some more examples of numbers that have been **completely factored**:

- $20 = 2 \cdot 2 \cdot 5$
- $100 = 2 \cdot 2 \cdot 5 \cdot 5$
- $63 = 3 \cdot 3 \cdot 7$
- $300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$

The whole reason we will want to do this is that it will allow us to collect factors together to find squares:

- $20 = 2 \cdot 2 \cdot 5 = (2 \cdot 2) \cdot 5 = 2^2 \cdot 5$
- $100 = 2 \cdot 2 \cdot 5 \cdot 5 = (2 \cdot 2) \cdot (5 \cdot 5) = 2^2 \cdot 5^2$
- $63 = 3 \cdot 3 \cdot 7 = (3 \cdot 3) \cdot 7 = 3^2 \cdot 7$
- $300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 = (2 \cdot 2) \cdot 3 \cdot (5 \cdot 5) = 2^2 \cdot 3 \cdot 5^2$

Coming up with your own minigoals--Examples:

For each of the examples, we will identify our minigoals. For the following examples, we have not actually simplified the expression—rather, we have outlined the smaller goals we might need to meet in order to fully simplify the expression. There may be other smaller steps that need to be done as a part of each minigoal—we are not outlining all of those here. And there may be another sequence of minigoals that would also work—the set of minigoals chosen in the example may not be the only approach to take to correctly simplify the expression.

A. $(2x^4y^2)^3$

Minigoals:

a) **Rewrite the expression without parentheses**. For this I will need the identity $x^n = \underbrace{x \cdot \ldots \cdot x}_{n\text{−many times}}$ (which in this case I can write as $a^3 = a \cdot a \cdot a$ since the exponent is a 3—then I will just need to define $a = 2x^4y^2$), because the one thing that requires the parentheses is the exponent outside it.

b) **Reorder all multiplication so that "similar" factors are next to one another**. After I do the first minigoal, I can see that I will have $2x^4y^2$multiplied by itself three times—at that point I will be able to think of the whole expression as a sequence of multiplication, and I will want to use the property $ab = ba$ to reorder that multiplication until all of the 2s are next to each other, all the x^4s are next to each other, and all the y^2 are next to each other.

c) **Rewrite all exponents as multiplication.** After that, in order to find some way to simplify the x^4s and y^2s, I will need to use the definition of the exponent to write all of these out as multiplication—this will allow me to write all the multiplication of xs out as a single exponent and all the multiplication of ys out as a single exponent.

d) **Rewrite all multiplication as exponents, where possible.**

B. $(4x - 3)^2$

Minigoals:

a) **Rewrite the expression without parentheses**. For this I will need the identity $x^n = \underbrace{x \cdot \ldots \cdot x}_{n\text{−many times}}$ (which in this case I can write as $a^2 = a \cdot a$ since the exponent is a 2—then I will just need to define $a = 4x - 3$), because the one thing that requires the parentheses is the exponent outside it.

b) **Use each of the distributive properties to perform all multiplication, in order to rewrite without the parentheses**. After I do the first minigoal, I can see that I will have $(4x - 3)$ multiplied by itself—at that point I will have to rewrite all subtraction as adding a negative (using $a - b = a + -b$), and then I will need to use first the identity $a(b+c) = ab + ac$ to write the expression without the second set of parentheses, and then I will need to use the identity $(a+b)c = ac + bc$ to rewrite the expression without the first set of parentheses.

c) **Simplify each term.** After all the multiplication from the previous step, I will need to use the identity $ab = ba$ to reorder the factors in each term so that I can carefully simplify each term correctly.

d) **Use the distributive property to combine "like" terms.** After I finish simplifying each term, I see that I will end up with two terms with an x in them, so I will need to use the property $ac + bc = (a+b)c$ to rewrite them as a single term.

C. $\sqrt{45} - \sqrt{20}$

Minigoals:

a) **Factor each number under each radical completely**. This will allow me to break each number down into a product of many much smaller numbers (e.g. $45 = 3 \cdot 3 \cdot 5$ and $20 = 2 \cdot 2 \cdot 5$).

b) **Identify any squares under the radical (since this radical is a square root sign)**. After I do the first minigoal, I will have a series of numbers being multiplied together under each radical sign—this will allow me to identify every time I have two of the same numbers being multiplied together that can be rewritten as a square: $a \cdot a = a^2$.

c) **Rewrite each radical as a product of smaller radicals in order to isolate any squares under their own radical.** I know that I can use the identity $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ to take any product under the radical and break it up into a product of radicals. Once I know where the squares are, I can use this property to isolate each square under its own radical.

d) **Use the definition of the radical to simplify anything of the form $\sqrt{x^2} = x$.** Now that I have isolated each square under its own radical, I can rewrite just that part of the expression without the radical.

e) **Use the distributive property to combine "like" terms.** After I finish simplifying each term in the previous step, I see that I will end up with two terms with an $\sqrt{5}$ in them, so I will need to use the property $ac + bc = (a+b)c$ to rewrite them as a single term. I will need to rewrite the subtraction as adding a negative (using $a - b = a + -b$) in order to do this.

D. $\frac{6x^{-2}y^5}{15x^4y^{-8}}$ $(where\ x, y \neq 0)$

Minigoals:

a) **Rewrite the expression without negative exponents**. Because it is hard to think intuitively about negative exponents, we often want to rewrite them using the identity $x^{-n} = \frac{1}{x^n}$. We will need to do this twice, once in the numerate with $n = 2$ and once in the denominator with $x = y$ and $n = 8$. The result of this will be fractions inside the top and bottom of a larger fraction, which we will need to tackle in the next minigoal.

b) **Use properties of fractions to rewrite the expression so that there are no longer fractions inside fractions**. We will have a fraction in the numerator and a fraction in the denominator after the first minigoal, so then we will need to use the properties $\frac{\frac{a}{b}}{c} = \frac{a}{bc}$ and $\frac{a}{\frac{b}{c}} = \frac{ac}{b}$ to rewrite the expression as just one single fraction.

c) **Rewrite all exponents as multiplication and factor all numbers with the aim of later producing fractions of the form $\frac{a}{a}$.** At the end of the previous minigoal, we will have xs and ys in different parts of the fraction with different exponents. If we write them all out as multiplication (e.g. $x^4 = x \cdot x \cdot x \cdot x$) using the identity $x^n = \underbrace{x \cdot \ldots \cdot x}_{n-many\ times}$, then we can think about how to simplify the part of the expression that contains only numbers, only xs, or only ys. We can also think about factoring the numbers, since $\frac{6}{15}$ can be rewritten as $\frac{3 \cdot 2}{3 \cdot 5}$.

d) **Use the properties of fractions to rewrite the one big fraction as a product of smaller fractions, some of which have the form $\frac{a}{a}$, and replace each fraction of the form $\frac{a}{a}$ with 1.** We will need to use the identity $\frac{ab}{cd} = \frac{a}{c} \cdot \frac{b}{d}$ (or the more general form $\frac{x_1 \cdot \ldots \cdot x_n}{y_1 \cdot \ldots \cdot y_n} = \frac{x_1}{y_1} \cdot \ldots \cdot \frac{x_n}{y_n}$) to rewrite the one big fraction as the product of smaller fractions, including ones of the form $\frac{a}{a}$ which can then subsequently be replaced with 1.

E. $\frac{2x^3y^2 - xy^2}{xy}$

Minigoals:

a) **Use the properties of fractions to break this into two fractions**. First we will need to rewrite the subtraction as adding a negative (using the identity $a = b = a + -b$), and then we can use the property $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ to break this one larger fraction into two separate simpler fractions.

b) **Simplify each individual fraction—start by rewriting all exponents as multiplication with the aim of later producing fractions of the form $\frac{a}{a}$.** At the end of the previous minigoal, we will have xs and ys in different parts of each fraction with different exponents. If we write them all out as multiplication (e.g. $x^3 = x \cdot x \cdot x$) using the identity $x^n = \underbrace{x \cdot \ldots \cdot x}_{n-many\ times}$, then we can think about how to simplify the part of the expression that contains only numbers, only xs, or only ys.

c) **Use the properties of fractions to rewrite each of the two larger fractions as a product of smaller fractions, some of which have the form $\frac{a}{a}$, and replace each fraction of the form $\frac{a}{a}$ with 1.** We will need to use the identity $\frac{ab}{cd} = \frac{a}{c} \cdot \frac{b}{d}$ (or the more general form $\frac{x_1 \cdot \ldots \cdot x_n}{y_1 \cdot \ldots \cdot y_n} = \frac{x_1}{y_1} \cdot \ldots \cdot \frac{x_n}{y_n}$) to rewrite each of the larger fractions as the product of smaller fractions, including ones of the form $\frac{a}{a}$ which can then subsequently be replaced with 1.

Now you try! Simplify as much as possible, using ONLY the identities given in this project. For all problems with numbers under a radical, be sure to factor the numbers completely first.

While simplifying, **rewrite all negative exponents as positive exponents.**

- **A.** **First, outline the minigoals needed to simplify each expression. This should be at the top of each problem**
- **B.** Then, use the identities given in this project to accomplish each minigoal, until the expression is completely simplified. **Feel free to modify your original minigoals if you find that they need to be changed in order to simplify the expression.**

1. $(3a^3b^4)^3$ Minigoals:
2. $(2y-4)^2$ Minigoals:
3. $(x+3)(2x^2-3x-2)$ Minigoals:

4. $(5x^2 + 2x - 5) - (2x^2 - 3x - 2)$

Minigoals:

5. $(3x^2y^3 - 3xy^4) - (5x^2y^3 - 2xy^4)$

Minigoals:

6. $\sqrt{27} - \sqrt{12}$

Minigoals:

7. $8 + 2\sqrt{5} - 9 + 7\sqrt{20}$

Minigoals:

8. $\frac{\sqrt{35}\sqrt{10}}{\sqrt{7}}$

Minigoals:

9. $\sqrt{3}(\sqrt{6} + \sqrt{3})$

Minigoals:

10. $\left(\sqrt{3}-2\right)\left(2\sqrt{3}+1\right)$

Minigoals:

11. $\frac{6x^2y^4}{2x^5y^3}$

Minigoals:

12. $\frac{4x^3y^{-5}}{6x^{-2}y^6}$

Minigoals:

13. $\frac{\left(2x^{-2}y^5\right)^{-2}}{4x^4y^8}$

Minigoals:

14. $\frac{4a^2b^3-2ab}{2ab}$

Minigoals:

15. $\frac{6x^8-9x^5+15x^3}{-3x^3}$

Minigoals: