Project 5: Choosing which Identities to use Rewrite Expressions

Identifying structure inside expressions—which identities could be applied to an expression?

Before we can really use identities systematically to rewrite an expression or equation, we have to figure out which identities could even be applied to a particularly expression. This means we have to look at the **structure** of the expression and look for structures that are similar to those in the identities that we know.

For each of the following examples, we are given several identities to consider. For each expression, we should look for expressions or sub-expressions that have the form or structure of the left or the right-hand side of the given identities. For each expression, we will write down **every** identity that could be applied to the expression (or to a sub-expression), and will identify which parts of the expression would correspond to the variables in that identity.

Examples:

A) Consider the following expression: $\frac{3x^{-2}y^5}{9x^5y^8}$ (where $x, y, \neq 0$)

For each of the following identities, if the identity can be applied to the expression above (or any sub-expression) exactly as the expression currently is written (i.e. without first rewriting the expression to fit the structure of the identity), 1) circle that identity, 2) write out the sub-expression to which it could be applied, and then 3) write out what the variables in the identity would have to be equal to in order to apply that identity to the expression. If an identity can't be used on the above expression, cross it out, and explain why it can't be used.

i. $x^{-n} = \frac{1}{x^n}$ $(whenever\ x \neq 0)$ — x^{-2} has the form of the left side, so $x = x$, $n = 2$

~~ii. $a - b = a + -b$~~ There is no subtraction or addition in this expression.

iii. $x^n = \underbrace{x \cdot \cdots \cdot x}_{n-many\ times}$ $(whenever\ n\ is\ a\ positive\ whole\ number)$

Three parts of this expression have the form of the left side of this identity:
y^5 has the form of the left side, so $x = y$, $n = 5$
x^5 has the form of the left side, so $x = x$, $n = 5$
y^8 has the form of the left side, so $x = y$, $n = 8$

iv. $\frac{a}{b} \cdot \frac{c}{d} \cdot \frac{e}{f} = \frac{ace}{bdf}$ $(whenever\ b, d, f \neq 0)$

$\frac{3x^{-2}y^5}{9x^5y^8}$ has the form of the right side, so $a = 3$, $b = 9$, $c = x^{-2}$, $d = x^5$, $e = y^5$, $f = y^8$

~~v. $\frac{a}{d} + \frac{b}{d} + \frac{c}{d} = \frac{a+b+c}{d}$ (whenever $b \neq 0$)~~ There is no addition in this expression.

1. Consider the following expression: $(2x+3)(3x+1)$

   For each of the following identities, if the identity can be applied to the expression above (or any sub-expression) exactly as the expression currently is written (i.e. without first rewriting the expression to fit the structure of the identity), 1) circle that identity, 2) write out the sub-expression to which it could be applied, and then 3) write out what the variables in the identity would have to be equal to in order to apply that identity to the expression. If an identity can't be used on the above expression, cross it out.

   i. $a(b+c)=ab+ac$

   ii. $(a+b)c=ac+bc$

   iii. $(a+b)+(c+d)=a+b+c+d$

2. Consider the following expression: $7x^3+2x^3-3x^2+2x+4x^2+9x$

   For each of the following identities, if the identity can be applied to the expression above (or any sub-expression) exactly as the expression currently is written (i.e. without first rewriting the expression to fit the structure of the identity), 1) circle that identity, 2) write out the sub-expression to which it could be applied, and then 3) write out what the variables in the identity would have to be equal to in order to apply that identity to the expression. If an identity can't be used on the above expression, cross it out.

   i. $a-b=a+-b$

   ii. $(a+b)c=ac+bc$

   iii. $a+b=b+a$

3. Consider the following expression: $\frac{1}{(3xy)^5} \cdot \frac{2xy^3}{3}$

For each of the following identities, if the identity can be applied to the expression above (or any sub-expression) exactly as the expression currently is written (i.e. without first rewriting the expression to fit the structure of the identity), 1) circle that identity, 2) write out the sub-expression to which it could be applied, and then 3) write out what the variables in the identity would have to be equal to in order to apply that identity to the expression. If an identity can't be used on the above expression, cross it out.

i. $x^{-n} = \frac{1}{x^n} \quad (\text{whenever } x \neq 0)$

ii. $ab = ba$

iii. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad (\text{whenever } b, d \neq 0)$

iv. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad (\text{whenever } c \neq 0)$

v. $x^n = \underbrace{x \cdot \cdots \cdot x}_{n\text{-many times}} \quad (\text{whenever } n \text{ is a positive whole number})$

4. Consider the following expression: $\sqrt{2x^4y^3} \cdot \sqrt{2y}$

Both of the following identities can be used to rewrite the expression above. For each of these identities, find 1) at least one sub-expression in the expression above that has the structure of the left side of the identity , and 2) at least one sub-expression that has the structure of the right side of the identity. For each of these sub-expressions, write out the sub-expression to which that side of the identity could be applied, and then 3) write out what the variables in the identity would have to be equal to in order to apply that identity to the expression.

i. $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

ii. $ab = ba$

Identities that could be applied to an expression if we rewrite it first

When we are rewriting expressions or equations, it is often with a particular end goal in mind. So usually we are not just interested in what identities can be applied to an expression in its current form, but also in what identities could be applied to the expression in a future step, if we take other steps to rewrite the expression first.

In this next set of examples, we will be given a particular identity to apply to the expression (or a sub-expression) and we will need to 1) show how the expression must first we rewritten before the identity can be applied, and then 2) circle the part of the rewritten expression to which we can apply the identity and 3) write out what each variable in the identity must equal to in order to apply the identity to the expression. The very act of rewriting will require us to use other identities first before applying the given identity. Here is a list of other identities that we can use to rewrite expressions in this section, in order to put them in the form of the given identity in each question:

- $x = 1x$ and $-x = -1 \cdot x$
- $a - b = a + -b$
- $a + b = b + a$
- $ab = ba$
- $x^n = \underbrace{x \cdot \ldots \cdot x}_{n-many\ times}$
- $\frac{a}{b} = a \cdot \frac{1}{b} \quad (whenever\ b \neq 0)$
- $\frac{ab}{c} = a \cdot \frac{b}{c} \quad (whenever\ c \neq 0)$
- $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad (whenever\ c \neq 0)$
- $\sqrt{x}\sqrt{y} = \sqrt{xy}$

Examples:

B) We want to use the identity $a(b + c) = ab + ac$ to rewrite the following expression:

$$(6x^2 - 5) - (2x^2 - 6)$$

i. First use other identities to rewrite this expression so that some part of the expression now has the form of the left side of the identity above.

We notice that the $-(2x^2 - 6)$ could be re-written in the form $a(b + c)$ if we first rewrite the subtraction in the middle as addition and we insert the coefficient 1 before the $(2x^2 - 6)$. So, we need the two identities: 1) $x = 1x$ and 2) $a - b = a + -b$ first:

$(6x^2 - 5) - (2x^2 - 6) = (6x^2 - 5) - 1(2x^2 - 6)$	Because $(2x^2 - 6) = 1(2x^2 - 6)$, using the identity $x = 1x$ and letting $x = (2x^2 - 6)$.
$(6x^2 - 5) - 1(2x^2 - 6) = (6x^2 - 5) + -1(2x^2 - 6)$	Using the identity $a - b = a + -b$ on the whole expression resulting from the previous step and letting $a = (6x^2 - 5)$, $b = 1(2x^2 - 6)$.
$(6x^2 - 5) + -1(2x^2 - 6) = (6x^2 - 5) + -1(2x^2 + -6)$	Because $2x^2 - 6 = 2x^2 + -6$, using the identity $a - b = a + -b$ and letting $a = 2x^2, b = 6$.

ii. Then, circle the part of the rewritten expression that has the form of the left side of the identity above.

The result of our rewriting is: $(6x^2 - 5) + -1(2x^2 + -6)$. The part $-1(2x^2 + -6)$ now has the form of the left side of the given identity $a(b + c) = ab + ac$, so circling that part:

$$(6x^2 + -5) + \boxed{-1(2x^2 + -6)}$$

iii. And finally, write out what each variable in the identity above must be equal to in order to apply that identity to the re-written expression.

So if $-1(2x^2 + -6)$ has the form of the left side of the given identity $a(b + c) = ab + ac$, that gives $a = -1,\ b = 2x^2,\ c = -6$. (We could now use the original identity to rewrite this expression.)

5. We want to use the identity $a(b+c) = ab + ac$ to rewrite the following expression:

$$(2x+3) - (3x-2)$$

i. First use other identities to rewrite this expression so that some part of the expression now has the form of the left side of the identity above.

ii. Then, circle the part of the rewritten expression that has the form of the left side of the identity above.

iii. And finally, write out what each variable in the identity above must be equal to in order to apply that identity to the re-written expression.

6. We want to use the identity $a(b+c) = ab + ac$ to rewrite the following expression:

$$(2x+3)(3x-2)$$

i. First use other identities to rewrite this expression so that some part of the expression now has the form of the left side of the identity above.

ii. Then, circle the part of the rewritten expression that has the form of the left side of the identity above.

iii. And finally, write out what each variable in the identity above must be equal to in order to apply that identity to the re-written expression.

7. We want to use the identity $(a+b)c = ac + bc$ to rewrite the following expression:

$$3x^2 - 2x + 5x^2 - 5$$

i. First use other identities to rewrite this expression so that some part of the expression now has the form of the right side of the identity above.

ii. Then, circle the part of the rewritten expression that has the form of the left side of the identity above.

iii. And finally, write out what each variable in the identity above must be equal to in order to apply that identity to the re-written expression.

8. We want to use the identity $x^{-n} = \frac{1}{x^n}$ $(whenever\ x \neq 0)$ to rewrite the following expression so that it no longer has a fraction form: $\frac{2x^2}{y^3}$

i. First use <u>other</u> identities to rewrite this expression so that some part of the expression now has the form of the <u>right</u> side of the identity above.

ii. Then, circle the part of the rewritten expression that has the form of the right side of the identity above.

iii. And finally, write out what each variable in the identity above must be equal to in order to apply that identity to the re-written expression.

9. We want to use the identity $\frac{x}{x} = 1$ $(whenever\ x \neq 0)$ to rewrite the following expression:

$$\frac{6x^2 - 3x}{3x}$$

i. First use <u>other</u> identities to rewrite this expression so that some part of the expression now has the form of the <u>left</u> side of the identity above.

ii. Then, circle the part of the rewritten expression that has the form of the left side of the identity above.

iii. And finally, write out what each variable in the identity above must be equal to in order to apply that identity to the re-written expression.

10. We want to use the identity $\sqrt{x^2} = x$ $(whenever\ x \geq 0)$ to rewrite the following expression:

$$\sqrt{xy} \cdot \sqrt{xy}$$

i. First use <u>other</u> identities to rewrite this expression so that some part of the expression now has the form of the <u>left</u> side of the identity above.

ii. Then, circle the part of the rewritten expression that has the form of the left side of the identity above.

iii. And finally, write out what each variable in the identity above must be equal to in order to apply that identity to the re-written expression.