Project 4: Using Identities to Rewrite Parts of Expressions

So far we have only used identities to rewrite expressions when the whole expression took the form of either the right or the left side of the identity. However, we can 1) use an identity to rewrite just **part of an expression or equation**, and 2) **we can use more than one identity to rewrite an expression**, using first one identity and then another to rewrite an expression or equation in several steps. In fact, any time in the past that you have simplified or rewritten an expression, or solved or rewritten an equation, you have already been doing this—you probably just weren't explicitly aware that this is what you were doing.

Step 1: Identifying which PART of an expression has the same structure as an identity

Before we can use identities to rewrite part of an expression or equation, we have to practice identifying which parts of an expression have the structure of the right or the left side of the identity that we are interested in. Otherwise we have no idea how to use the identity to rewrite the expression.

For each of the following examples, simply identify one part of the expression that has the form of either the right or the left side of the given identity (there may be more than one correct answer).

Examples:

A) Circle some part of the expression below that has the structure of the left or the right side of the identity $\boldsymbol{a(b+c)=ab+ac}$ and identify exactly which parts of the expression would be equal to $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$:

$$\boldsymbol{(3x-2)+-3(x+7)}$$

Where in this expression can we see something that looks like either the left or the right side of the identity above? The second half of this expression, $-3(x+7)$, has exactly the form of the left side of the identity, so we circle this:

$$(3x-2)+\boxed{-3(x+7)}$$

Now we identify which parts of this sub-expression $-3(x+7)$ correspond to a, b and c.

$a=-3$, because -3 is outside the parentheses to the left, and is being multiplied by the parentheses, just like a

$b=x$, because x is the first term inside the parentheses, just like b

$c=7$, because 7 is the second term inside the parentheses, just like c

B) Circle some part of the expression below that has the structure of the left or the right side of the identity $\boldsymbol{x^{-n}=\frac{1}{x^n}}$ (whenever $\boldsymbol{x\neq 0}$) and identify exactly which parts of the expression would be equal to $\boldsymbol{x}$ and $\boldsymbol{n}$:

$$\boldsymbol{5x(2x^2y)^{-3} \qquad (2x^2y\neq 0)}$$

Where in this expression can we see something that looks like either the left or the right side of the identity above? This part: $(2x^2y)^{-3}$ has exactly the form of the left side of the identity, so we circle this:

Now we identify which parts of this sub-expression $(2x^2y)^{-3}$ correspond to x and n.

$x=2x^2y$, because this is the base—the whole part that is being raised to the negative exponent

$n=3$, because this is the value of the exponent that comes after the negative sign

1. Circle some part of the expression below that has the structure of the left or the right side of the identity $x^{-n}=\frac{1}{x^n}$ (whenever $x\neq 0$) and identify exactly which parts of the expression would be equal to x and n:

$$\frac{3x^4}{4x^2y^{-3}z^7}$$

2. Circle some part of the expression below that has the structure of the left or the right side of the identity $ba + ca = (b + c)a$ and identify exactly which parts of the expression would be equal to a, b and c:

$$4x^3 + 3x^2 + 8x^2 + 8x - 1$$

3. Circle some part of the expression below that has the structure of the left or the right side of the identity $ab = ba$ and identify exactly which parts of the expression would be equal to a and b. Choose a different part for each of the following criteria:

$$2x \cdot 3yx$$

   a. Choose a part that, after you apply the identity, will result in 2 and 3 being next to each other.

   b. Choose a part that, after you apply the identity, will result in the two x's being next to each other.

   c. Choose a different part from a. or b. above that, after you apply the identity, will put the 2 and the 3 next to one another.

Step 2: Replacing only PART of an expression with another equivalent part

In order to rewrite part of an expression, we need to practice the idea that **I can replace any sub-expression** in a larger expression **with anything that is equivalent to the original sub-expression**. Again, this is something that you have always done when simplifying or rewriting expressions, or when solving or rewriting equations, but you probably haven't been explicitly aware of this in the past.

Examples:

C) Use this equation $-3(x+7) = -3x + -21$ to replace the original expression with an equivalent expression that has a different form:

$$\mathbf{(3x - 2) + -3(x + 7)}$$

In this expression can we can replace $-3(x + 7)$ with $-3x + -21$. We start by circling what we want to replace:

$(3x - 2) + \boxed{-3(x + 7)}$

We replace the part that we want to replace with parentheses, and we recopy everything else in the expression exactly as it is:

$(3x - 2) + \overbrace{(\qquad\qquad)}^{-3(x+7)}$

And now we replace the $-3(x + 7)$ with its equivalent $-3x + -21$:

$(3x - 2) + \overbrace{(-3x + -21)}^{-3(x+7)}$

So $(3x - 2) + \boxed{-3(x + 7)} = (3x - 2) + \boxed{(-3x + -21)}$

D) Use the equation $(2x^2y)^{-3} = \frac{1}{(2x^2y)^3}$ (whenever $x \neq 0$) to replace the original expression with an equivalent expression that has a different form:

$$\mathbf{5x(2x^2y)^{-3} \qquad (2x^2y \neq 0)}$$

In this expression can we can replace $(2x^2y)^{-3}$ with $\frac{1}{(2x^2y)^3}$. We start by circling what we want to replace:

$5x\boxed{(2x^2y)^{-3}}$

We replace the part that we want to replace with parentheses, and we recopy everything else in the expression exactly as it is:

$5x\overbrace{(\qquad)}^{(2x^2y)^{-3}}$

And now we replace the $(2x^2y)^{-3}$ with its equivalent $\frac{1}{(2x^2y)^3}$:

$5x\overbrace{\left(\frac{1}{(2x^2y)^3}\right)}^{(2x^2y)^{-3}}$

So $5x\boxed{(2x^2y)^{-3}} = 5x\boxed{\frac{1}{(2x^2y)^3}}$

4. Use the equation $y^{-3} = \frac{1}{y^3}$ (whenever $x \neq 0$) to replace the original expression with an equivalent expression that has a different form:

$$\frac{3x^4}{4x^2y^{-3}z^7}$$

5. Use the equation $3x^2 + 8x^2 = (3+8)x^2$ to replace the original expression with an equivalent expression that has a different form:

$$4x^3 + 3x^2 + 8x^2 + 8x - 1$$

6. Use the equations $x \cdot 3 = 3 \cdot x$ and $yx = xy$ to replace the original expression with an equivalent expression that has a different form (you may want to do this in two separate steps):

$$2x \cdot 3yx$$

Step 3: Using an identity to rewrite PART of an expression

For each of the following examples, we use an identity to rewrite **part** of an expression or equation. When we do this, **we have to copy all of the other parts of the expression or equation exactly the same way from one step to**

the next, and **the only thing that we then change is the part that looks like one side of an identity.** This will be easier to understand by looking at some specific examples.

Examples:

E) Use the identity $a(b+c) = ab + ac$ to replace the original expression with an equivalent expression that has a different form:

$$(3x-2) + -3(x+7)$$

Where in this expression can we see something that looks like either the left or the right side of the identity above? The second half of this expression, $-3(x+7)$, has exactly the form of the left side of the identity—this is the part that we can rewrite.
So for $-3(x+7)$:
$a = -3, \;\; b = x, \;\; c = 7$

$$a(b+c) = ab + ac \quad \rightarrow \quad \overbrace{(\;\;)}^{a}\left(\overbrace{(\;\;)}^{b} + \overbrace{(\;\;)}^{c}\right) = \overbrace{(\;\;)}^{a}\overbrace{(\;\;)}^{b} + \overbrace{(\;\;)}^{a}\overbrace{(\;\;)}^{c}$$

If we just wanted to rewrite $-3(x+7)$, we would write only this: $\overbrace{(-3)}^{a}\left(\overbrace{(x)}^{b} + \overbrace{(7)}^{c}\right) = \overbrace{(-3)}^{a}\overbrace{(x)}^{b} + \overbrace{(-3)}^{a}\overbrace{(7)}^{c}$

However, we need to rewrite the **whole** original expression, so we can't leave out the first term. So once we have figured out how to rewrite $-3(x+7)$, we have to go back to the original expression like this:

$$(3x-2) + \boxed{(-3(x+7))} = (3x-2) + \boxed{\big((-3)(x) + (-3)(7)\big)}$$

So $\boxed{\mathbf{(3x-2) + -3(x+7) = (3x-2) + \big((-3)(x) + (-3)(7)\big)}}$

F) Use the identity $x^{-n} = \frac{1}{x^n}$ (whenever $x \neq 0$) to replace the original expression with an equivalent expression that has a different form:

$$5x\left(2x^2y\right)^{-3} \qquad (2x^2y \neq 0)$$

This part: $(2x^2y)^{-3}$ has exactly the form of the left side of the identity—this is the part of we can rewrite.
So for $(2x^2y)^{-3}$:
$x = 2x^2y, \;\; n = 3$

$$\frac{1}{x^n} = x^{-n} \quad \rightarrow \quad \overbrace{(\;\;)}^{x}{}^{-\overbrace{(\;\;)}^{n}} = \frac{1}{\overbrace{(\;\;)}^{x}{}^{\overbrace{(\;\;)}^{n}}}$$

$$\overbrace{(2x^2y)}^{x}{}^{-\overbrace{(3)}^{n}} = \frac{1}{\overbrace{(2x^2y)}^{x}{}^{\overbrace{(3)}^{n}}}$$ (We know that $x \neq 0$ because $2x^2y \neq 0$.)

If we just wanted to rewrite $(2x^2y)^{-3}$, we would write only this: $\overbrace{(2x^2y)}^{x}{}^{-\overbrace{(3)}^{n}} = \frac{1}{\overbrace{(2x^2y)}^{x}{}^{\overbrace{(3)}^{n}}}$

However, we need to rewrite the **whole** original expression, so we can't leave out the first term. So once we have figured out how to rewrite $(2x^2y)^{-3}$, we have to go back to the original expression like this:

$$5x\boxed{\left((2x^2y)^{-3}\right)} = 5x\boxed{\left(\frac{1}{\overbrace{(2x^2y)}^{x}{}^{\overbrace{(3)}^{n}}}\right)}$$

So $\boxed{\mathbf{5x\left(2x^2y\right)^{-3} = 5x\left(\frac{1}{\overbrace{(2x^2y)}^{x}{}^{\overbrace{(3)}^{n}}}\right)}}$

Now you try! For each of the following problems, use the given identity to rewrite the given expression in a different equivalent form. For each of these questions, you will only be able to use the identity on **part** of the expression, but be sure to rewrite the **whole** expression, recopying each time any parts that don't change. **Use the examples above as a model, writing out all of the same steps.**

7. Use the identity $x^{-n} = \frac{1}{x^n}$ (whenever $x \neq 0$) to replace the original expression with an equivalent expression that has a different form:

$$\frac{3x^4}{4x^2y^{-3}z^7}$$

8. Use the identity $ba + ca = (b + c)a$ to replace the original expression with an equivalent expression that has a different form:

$$4x^3 + 3x^2 + 8x^2 + 8x - 1$$

9. Use the identity $ab = ba$ to replace the original expression with an equivalent expression that has a different form and to put all the numbers next to each other and to put all the same variables next to each other. You will need to use this identity in more than one place to accomplish this:

$$2x \cdot 3yx$$

10. Use the identity $x = 1x$ to replace the original expression with an equivalent expression that has a different form—specifically, use this identity to rewrite the first x^2 in this expression:

$$\sqrt{x^2 + 3x^2}$$

Step 4: Using several identities to rewrite part or all of an expression in MULTIPLE STEPS

In addition to (and often alongside) rewriting just part of an expression or equation, we may also need to apply several different identities in multiple steps in order to rewrite an expression the way that we want.

Examples:

G) Use the identities 1) $\boldsymbol{x = 1x}$ to rewrite the second $\boldsymbol{x^2}$ in the expression and 2) $\boldsymbol{a - b = a + -b}$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\boldsymbol{8x^2 - x^2}$$

<u>First identity:</u> Since an x appears in both the identity and in the expression, we may first want to rewrite the identity with a different letter. Let's pick z. Then $x = 1x \quad \rightarrow z = 1z$. And to use this to rewrite x^2, we would do this:

$z = x^2$

$z = 1z$

$\overbrace{(\quad)}^{z} = 1\overbrace{(\quad)}^{z} \quad \rightarrow \quad \overbrace{(x^2)}^{z} = 1\overbrace{(x^2)}^{z}$

So the **whole** expression would then be: $8x^2 - (x^2) = 8x^2 - (1x^2)$

Now we need to use the identity $a - b = a + -b$ on this NEW expression. The expression $8x^2 - (1x^2)$ looks like the left half of this identity.

$a = 8x^2,\ \ b = (1x^2)$

$\overbrace{(\quad)}^{a} - \overbrace{(\quad)}^{b} = \overbrace{(\quad)}^{a} + -\overbrace{(\quad)}^{b} \quad \rightarrow \quad \overbrace{(8x^2)}^{a} - \overbrace{(1x^2)}^{b} = \overbrace{(8x^2)}^{a} + -\overbrace{(1x^2)}^{b}$

So $\boxed{\boldsymbol{8x^2 - x^2 = 8x^2 + -1x^2}}$

H) Use the identities 1) $\boldsymbol{a - b = a + -b}$ and 2) $\boldsymbol{a(b + c) = ab + ac}$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\boldsymbol{3(2x - 5)}$$

<u>First identity:</u> This part of the expression: $2x - 5$ has the form of the left side of the identity $a - b = a + -b$.

So $a = 2x,\ \ b = 5$

$a - b = a + -b \quad \rightarrow \quad \overbrace{(\quad)}^{a} - \overbrace{(\quad)}^{b} = \overbrace{(\quad)}^{a} + -\overbrace{(\quad)}^{b} \quad \rightarrow \quad \overbrace{(2x)}^{a} - \overbrace{(5)}^{b} = \overbrace{(2x)}^{a} + -\overbrace{(5)}^{b}$

So the **whole** expression would be: $3(2x - 5) = 3(2x + -5)$.

<u>Second identity:</u> The NEW expression $3(2x + -5)$ has the form of the left side of the identity $a(b + c) = ab + ac$.

So $a = 3,\ \ b = 2x,\ \ c = -5$

So $a(b + c) = ab + ac \quad \rightarrow \quad \overbrace{(\quad)}^{a}\left(\overbrace{(\quad)}^{b} + \overbrace{(\quad)}^{c}\right) = \overbrace{(\quad)}^{a}\overbrace{(\quad)}^{b} + \overbrace{(\quad)}^{a}\overbrace{(\quad)}^{c}$

$\rightarrow \quad \overbrace{(3)}^{a}\left(\overbrace{(2x)}^{b} + \overbrace{(-5)}^{c}\right) = \overbrace{(3)}^{a}\overbrace{(2x)}^{b} + \overbrace{(3)}^{a}\overbrace{(-5)}^{c}$

So $\boxed{\boldsymbol{3(2x - 5) = (3)(2x) + (3)(-5)}}$

I) Use the identities 1) $\boldsymbol{a - b = a + -b}$, 2) $\boldsymbol{a(b + c) = ab + ac}$, and 3) $\boldsymbol{(b + c)a = ba + ca}$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\boldsymbol{(3x - 2)(4x - 1)}$$

<u>First identity:</u> Both $(3x - 2)$ and $(4x - 1)$ are of the form of the left side of the identity $a - b = a + -b$. So they can both be rewritten.

For $(3x - 2)$, $a = 3x,\ \ b = 2$

So $a - b = a + -b \quad \rightarrow \quad \overbrace{(\quad)}^{a} - \overbrace{(\quad)}^{b} = \overbrace{(\quad)}^{a} + -\overbrace{(\quad)}^{b} \quad \rightarrow \quad \overbrace{(3x)}^{a} - \overbrace{(2)}^{b} = \overbrace{(3x)}^{a} + -\overbrace{(2)}^{b}$

For $(4x - 1)$, $a = 4x,\ \ b = 1$

So $a - b = a + -b \quad \rightarrow \quad \overbrace{(\quad)}^{a} - \overbrace{(\quad)}^{b} = \overbrace{(\quad)}^{a} + -\overbrace{(\quad)}^{b} \quad \rightarrow \quad \overbrace{(4x)}^{a} - \overbrace{(1)}^{b} = \overbrace{(4x)}^{a} + -\overbrace{(1)}^{b}$

So the **whole** expression would be: $(3x - 2)(4x - 1) = (3x + -2)(4x + -1)$.

<u>Second identity:</u> The NEW expression $(3x + -2)(4x + -1)$ has the form of the left side of the identity $a(b + c) = ab + ac$ if we define a, b and c this way:

$a = (3x + -2),\ \ b = 4x,\ \ c = -1$

So $a(b+c)=ab+ac \quad \rightarrow \quad \overbrace{(\quad)}^{a}\left(\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{c}\right)=\overbrace{(\quad)}^{a}\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{a}\overbrace{(\quad)}^{c}$

$\rightarrow \quad \overbrace{(3x+-2)}^{a}\left(\overbrace{(4x)}^{b}+\overbrace{(-1)}^{c}\right)=\overbrace{(3x+-2)}^{a}\overbrace{(4x)}^{b}+\overbrace{(3x+-2)}^{a}\overbrace{(-1)}^{c}$

So $(3x+-2)(4x+-1)=(3x+-2)(4x)+(3x+-2)(-1)$

Third identity: Both $(3x+-2)(4x)$ and $(3x+-2)(-1)$ in the NEW expression have the form of the left side of the identity $(b+c)a=ba+ca$.

For $(3x+-2)(4x)$, $a=4x,\ \ b=3x,\ \ c=-2$

So $(b+c)a=ba+ca \quad \rightarrow \quad \left(\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{c}\right)\overbrace{(\quad)}^{a}=\overbrace{(\quad)}^{b}\overbrace{(\quad)}^{a}+\overbrace{(\quad)}^{c}\overbrace{(\quad)}^{a}$

$\rightarrow \quad \left(\overbrace{(3x)}^{b}+\overbrace{(-2)}^{c}\right)\overbrace{(4x)}^{a}=\overbrace{(3x)}^{b}\overbrace{(4x)}^{a}+\overbrace{(-2)}^{c}\overbrace{(4x)}^{a}$

For $(3x+-2)(-1)$, $a=-1,\ \ b=3x,\ \ c=-2$

So $(b+c)a=ba+ca \quad \rightarrow \quad \left(\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{c}\right)\overbrace{(\quad)}^{a}=\overbrace{(\quad)}^{b}\overbrace{(\quad)}^{a}+\overbrace{(\quad)}^{c}\overbrace{(\quad)}^{a}$

$\rightarrow \quad \left(\overbrace{(3x)}^{b}+\overbrace{(-2)}^{c}\right)\overbrace{(-1)}^{a}=\overbrace{(3x)}^{b}\overbrace{(-1)}^{a}+\overbrace{(-2)}^{c}\overbrace{(-1)}^{a}$

So the **whole** expression becomes:

$(3x+-2)(4x)+(3x+-2)(-1)=(3x)(4x)+(-2)(4x)+(3x)(-1)+(-2)(-1)$

So $\boxed{\mathbf{(3x-2)(4x-1)=(3x)(4x)+(-2)(4x)+(3x)(-1)+(-2)(-1)}}$

J) Use the identities 1) $x=1x$ (using the whole second set of parentheses as the x for this identity), 2) $a-b=a+-b$, and 3) $a(b+c)=ab+ac$ (on the second set of parentheses only) to replace the original expression with an equivalent expression that has a different form.

$$\mathbf{(2x^2-5)-(x^2+2)}$$

First identity: Because both our identity and our expression have x's in them, we might want to rewrite the identity using a different variable for x, in order to avoid confusion. Let's use a instead:

$x^{-n}=\frac{1}{x^n}$ (whenever $x\neq 0$) $\quad \rightarrow \quad a=1a$

This part of the expression (x^2+2) is the part that we need to treat as the a in $a=1a$.

So $a=x^2+2$

So $a=1a \quad \rightarrow \quad \overbrace{(\quad)}^{a}=1\overbrace{(\quad)}^{a} \rightarrow \quad \overbrace{(x^2+2)}^{a}=1\overbrace{(x^2+2)}^{a}$

So $x^2+2=1(x^2+2)$ and the **whole** expression becomes: $(2x^2-5)-(x^2+2)=(2x^2-5)-1(x^2+2)$

Second identity: The NEW expression $(2x^2-5)-1(x^2+2)$ has the form of the left side of the identity $a-b=a+-b$

So $a=(2x^2-5),\ \ b=1(x^2+2)$

So $a-b=a+-b \quad \rightarrow \quad \overbrace{(\quad)}^{a}-\overbrace{(\quad)}^{b}=\overbrace{(\quad)}^{a}+-\overbrace{(\quad)}^{b} \quad \rightarrow \quad \overbrace{(2x^2-5)}^{a}-\overbrace{(1(x^2+2))}^{b}=\overbrace{(2x^2-5)}^{a}+$
$-\overbrace{(1(x^2+2))}^{b}$

So $(2x^2-5)-1(x^2+2)=(2x^2-5)+-1(x^2+2)$

Third identity: The expression $-1(x^2+2)$ is the part of the NEW expression that has the form of the left side of the identity $a(b+c)=ab+ac$.

So $a=-1,\ \ b=x^2,\ \ c=2$

So $a(b+c)=ab+ac \quad \rightarrow \quad \overbrace{(\quad)}^{a}\left(\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{c}\right)=\overbrace{(\quad)}^{a}\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{a}\overbrace{(\quad)}^{c}$

$\rightarrow \quad \overbrace{(-1)}^{a}\left(\overbrace{(x^2)}^{b}+\overbrace{(2)}^{c}\right)=\overbrace{(-1)}^{a}\overbrace{(x^2)}^{b}+\overbrace{(-1)}^{a}\overbrace{(2)}^{c}$

So $-1(x^2+2)=(-1)(x^2)+(-1)(2)$ and the **whole** expression becomes:

$(2x^2 - 5) + -1(x^2 + 2) = (2x^2 - 5) + (-1)(x^2) + (-1)(2)$

So $\boxed{\boldsymbol{(2x^2 - 5) - (x^2 + 2) = (2x^2 - 5) + (-1)(x^2) + (-1)(2)}}$

K) Use the identity $x^{-n} = \frac{1}{x^n}$ (whenever $x \neq 0$) twice in a row to replace the original expression with an equivalent expression that has a different form.

$$\mathbf{3(2x^{-5}y)^{-4} \qquad 2x^{-5}y, x \neq 0}$$

<u>First time:</u> Because both our identity and our expression have x's in them, we might want to rewrite the identity using a different variable for x, in order to avoid confusion. Let's use a instead:

$x^{-n} = \frac{1}{x^n}$ (whenever $x \neq 0$) $\rightarrow$ $a^{-n} = \frac{1}{a^n}$ (whenever $a \neq 0$)

This part of the expression $(2x^{-5}y)^{-4}$ is the part that has the form of the left side of the identity $a^{-n} = \frac{1}{a^n}$.

So $a = 2x^{-5}y,\ \ n = 4$

So $a^{-n} = \frac{1}{a^n}$ $\rightarrow$ $\overbrace{(\ )}^{a}{}^{-\overbrace{(\ )}^{n}} = \frac{1}{\overbrace{(\ )}^{a}{}^{\overbrace{(\ )}^{n}}}$ $\rightarrow$ $\overbrace{(2x^{-5}y)}^{a}{}^{-\overbrace{(4)}^{n}} = \frac{1}{\overbrace{(2x^{-5}y)}^{a}{}^{\overbrace{(4)}^{n}}}$ (because $2x^{-5}y \neq 0$, we have $a \neq 0$)

So $3\boxed{((2x^{-5}y)^{-4})} = 3\boxed{\left(\frac{1}{(2x^{-5}y)^4}\right)}$

<u>Second time:</u> Again we will use the identity with a instead of x to avoid confusion. Now it is the x^{-5} part of the NEW expression that has the form of the left side of the identity $a^{-n} = \frac{1}{a^n}$:

So $a = x,\ \ n = 5$

So $a^{-n} = \frac{1}{a^n}$ $\rightarrow$ $\overbrace{(\ )}^{a}{}^{-\overbrace{(\ )}^{n}} = \frac{1}{\overbrace{(\ )}^{a}{}^{\overbrace{(\ )}^{n}}}$ $\rightarrow$ $\overbrace{(x)}^{a}{}^{-\overbrace{(5)}^{n}} = \frac{1}{\overbrace{(x)}^{a}{}^{\overbrace{(5)}^{n}}}$ (because $x \neq 0$, we have $a \neq 0$)

So $3\left(\frac{1}{\left(2\boxed{(x^{-5})}y\right)^4}\right) = 3\left(\frac{1}{\left(2\boxed{\left(\frac{1}{x^5}\right)}y\right)^4}\right)$

So $\boxed{\mathbf{3(2x^{-5}y)^{-4} = 3\left(\frac{1}{\left(2\left(\frac{1}{x^5}\right)y\right)^4}\right)}}$

L) Use the identities 1) $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ (whenever $c \neq 0$), 2) $\frac{x}{x} = 1$ (whenever $x \neq 0$), 3) $(a+b)+c = a+(b+c)$, 4) $x + -x = 0$, and 5) $x + 0 = x$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\mathbf{\frac{x+y}{y} + -1 \qquad (y \neq 0)}$$

<u>First identity:</u> The $\frac{x+y}{y}$ part of the expression has the form of the left side of the first identity $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.

So $a = x,\ b = y,\ c = y$ (and since $y \neq 0$, we have $c \neq 0$)

So $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ $\rightarrow$ $\frac{\overbrace{(\ )}^{a}+\overbrace{(\ )}^{b}}{\underbrace{(\ )}_{c}} = \frac{\overbrace{(\ )}^{a}}{\underbrace{(\ )}_{c}} + \frac{\overbrace{(\ )}^{b}}{\underbrace{(\ )}_{c}}$ $\rightarrow$ $\frac{\overbrace{(x)}^{a}+\overbrace{(y)}^{b}}{\underbrace{(y)}_{c}} = \frac{\overbrace{(x)}^{a}}{\underbrace{(y)}_{c}} + \frac{\overbrace{(y)}^{b}}{\underbrace{(y)}_{c}}$

So we can rewrite $\boxed{\left(\frac{x+y}{y}\right)} + -1 = \boxed{\left(\frac{x}{y} + \frac{y}{y}\right)} + -1$

<u>Second identity:</u> The $\frac{y}{y}$ part of the NEW expression has the form of the left side of the second identity $\frac{x}{x} = 1$.

So $x = y$ (and since $y \neq 0$, we have $x \neq 0$)

So $\frac{x}{x} = 1$ $\rightarrow$ $\frac{\overbrace{(\ )}^{x}}{\underbrace{(\ )}_{x}} = 1$ $\rightarrow$ $\frac{\overbrace{(y)}^{x}}{\underbrace{(y)}_{x}} = 1$

So we can rewrite $\left(\frac{x}{y} + \boxed{\left(\frac{y}{y}\right)}\right) + -1 = \left(\frac{x}{y} + \boxed{(1)}\right) + -1$ which is just $\left(\frac{x}{y} + 1\right) + -1$

<u>Third identity:</u> The whole NEW expression now has the form of the left side of the identity $(a+b)+c = a+(b+c)$.

So $a = \frac{x}{y},\ \ b = 1,\ \ c = -1$

So $(a+b)+c = a+(b+c)$ $\rightarrow$ $\left(\overbrace{(\ )}^{a}+\overbrace{(\ )}^{b}\right)+\overbrace{(\ )}^{c}=\overbrace{(\ )}^{a}+\left(\overbrace{(\ )}^{b}+\overbrace{(\ )}^{c}\right)$

$\rightarrow$ $\left(\overbrace{\left(\frac{x}{y}\right)}^{a}+\overbrace{(1)}^{b}\right)+\overbrace{(-1)}^{c}=\overbrace{\left(\frac{x}{y}\right)}^{a}+\left(\overbrace{(1)}^{b}+\overbrace{(-1)}^{c}\right)$

So we can rewrite $\left(\frac{x}{y}+1\right)+-1=\frac{x}{y}+(1+-1)$

<u>Fourth identity:</u> This part of the NEW expression $(1+-1)$ now has the form of the left side of the identity $x+-x=0$

So $x=1$

So $x+-x=0$ $\rightarrow$ $\overbrace{(\ )}^{x}+-\overbrace{(\ )}^{x}=0$ $\rightarrow$ $\overbrace{(1)}^{x}+-\overbrace{(1)}^{x}=0$

So we can rewrite $\frac{x}{y}+\boxed{(1+-1)}=\frac{x}{y}+\boxed{(0)}$ which is just $\frac{x}{y}+0$

<u>Fifth identity:</u> The whole expression now has the form of the left side of the identity $x+0=x$. But since both the identity and the expression have x's in them, we may want to rewrite the identity using a different variable. Let's use c instead. Then $x+0=x$ $\rightarrow$ $c+0=c$

So $c=\frac{x}{y}$

So $c+0=c$ $\rightarrow$ $\overbrace{(\ )}^{c}+0=\overbrace{(\ )}^{c}$ $\rightarrow$ $\overbrace{\left(\frac{x}{y}\right)}^{c}+0=\overbrace{\left(\frac{x}{y}\right)}^{c}$

So we can rewrite $\frac{x}{y}+0=\frac{x}{y}$

And therefore $\boxed{\frac{x+y}{y}+-1=\frac{x}{y}}$ **(as long as $y \neq 0$)**

Now you try! For each of the following problems, use the given identities to rewrite the given expression in a different equivalent form. **Use the examples above as a model, writing out all of the same steps. Be sure to apply the identity from each step to the NEW expression that is the END result of the previous step** (do NOT simply apply the identity in steps 2 or later to the original expression).

11. Use the identities 1) $a(b+c)=ab+ac$ and 2) $(b+c)a=ba+ca$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$(2x+1)(x+5)$$

12. Use the identities 1) $x=1x$ (using the whole second set of parentheses as the x for this identity), 2) $a-b=a+-b$, and 3) $a(b+c+d)=ab+ac+ad$ (on the second set of parentheses only) to replace the original expression with an equivalent expression that has a different form.

$$(x^2+2x-1)-(3x^2-4x+7)$$

13. Use the identities 1) $\sqrt{ab}=\sqrt{a}\sqrt{b}$ and 2) $\sqrt{x^2}$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\sqrt{xy^2}$$

14. Use the identities 1) $ab = ba$ to reverse the order of the sub-expression x^2y at the top of the fraction, 2) $\frac{ab}{cd} = \frac{a}{c} \cdot \frac{b}{d}$ (whenever $c, d \neq 0$) to separate the x^2z from the other numbers and variables in both the top and bottom, 3) $\frac{x}{x} = 1$ (whenever $x \neq 0$) and 4) $x \cdot 1 = x$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\frac{2x^2yz}{3x^2z} \qquad (x, z \neq 0)$$