Project 4: Using Identities to Rewrite Parts of Expressions

So far we have only used identities to rewrite expressions when the whole expression took the form of either the right or the left side of the identity. However, we can 1) use an identity to rewrite just **part of an expression or equation**, and 2) **we can use more than one identity to rewrite an expression**, using first one identity and then another to rewrite an expression or equation in several steps. In fact, any time in the past that you have simplified or rewritten an expression, or solved or rewritten an equation, you have already been doing this—you probably just weren't explicitly aware that this is what you were doing.

Replacing PART of an expression with another equivalent part

Here is a key idea, that is necessary to do almost anything in algebra:

We can replace any sub-expression with an equivalent sub-expression using substitution. If we do this, the resulting whole expression will also be equivalent to the original expression.

Again, this is something that you have always done when simplifying or rewriting expressions, or when solving or rewriting equations, but you probably haven't been explicitly aware of this in the past. Just as with other substitutions, **we have to use parenthesis around the original subexpression, and substitute into those parentheses, in order to ensure that we don't accidentally change the surrounding structure.**

Examples:

A) Use this equation $-3(x+7) = -3x + -21$ to replace the original expression below with an equivalent expression that has a different form:

$$(3x-2) + -3(x+7)$$

In this expression can we can replace $-3(x+7)$ with $-3x + -21$. We start by circling what we want to replace:

$(3x-2) + \boxed{-3(x+7)}$

We replace the part that we want to replace with parentheses, and we recopy everything else in the expression exactly as it is:

$(3x-2) + \overbrace{(\qquad)}^{-3(x+7)}$

And now we replace the $-3(x+7)$ with its equivalent $-3x + -21$:

$(3x-2) + \overbrace{(-3x + -21)}^{-3(x+7)}$

So $(3x-2) + \boxed{-3(x+7)} = (3x-2) + \boxed{(-3x + -21)}$

B) Use the equation $(2x^2y)^{-3} = \frac{1}{(2x^2y)^3}$ (whenever $x \neq 0$) to replace the original expression with an equivalent expression that has a different form:

$$5x(2x^2y)^{-3} \qquad (2x^2y \neq 0)$$

In this expression can we can replace $(2x^2y)^{-3}$ with $\frac{1}{(2x^2y)^3}$. We start by circling what we want to replace:

$5x\boxed{(2x^2y)^{-3}}$

We replace the part that we want to replace with parentheses, and we recopy everything else in the expression exactly as it is:

$5x\overbrace{(\qquad)}^{(2x^2y)^{-3}}$

And now we replace the $(2x^2y)^{-3}$ with its equivalent $\frac{1}{(2x^2y)^3}$:

$5x\overbrace{\left(\frac{1}{(2x^2y)^3}\right)}^{(2x^2y)^{-3}}$

So $5x\boxed{(2x^2y)^{-3}} = 5x\boxed{\frac{1}{(2x^2y)^3}}$

Now you try! For each of the following expressions:
A) CIRCLE the sub-expression that is the same as one side of the equation; then
B) rewrite the expression with empty parentheses in place of that expression, and finally;
C) substitute in the equivalent sub-expression into those open parentheses to create an equivalent expression.

1. Use the equation $y^{-3} = \frac{1}{y^3}$ (whenever $x \neq 0$) to replace the original expression with an equivalent expression that has a different form:

$$\frac{3x^4}{4x^2y^{-3}z^7}$$

2. Use the equation $3x^2 + 8x^2 = (3+8)x^2$ to replace the original expression with an equivalent expression that has a different form:

$$4x^3 + 3x^2 + 8x^2 + 8x - 1$$

3. Use the equations $x \cdot 3 = 3 \cdot x$ and $yx = xy$ to replace the original expression with an equivalent expression that has a different form (you may want to do this in two separate steps):

$$2x \cdot 3yx$$

Creating an equivalent expression by using identities to perform substitution

Now let's use this idea (of substituting one equivalent sub-expression for another) to create equivalent expressions in a different form. We will have to do this in several steps:

Step 1: Identify **which part** of the expression has the form of the left or right of the identity and circle that part. **Check to be sure that the part you have circled can be treated as a SINGLE UNIT, according to the conventions and the order of operations.**

Step 2: Focus temporarily on rewriting **just that sub-expression** by using the identity to replace it with an equivalent sub-expression.

Step 3: Substitute the new equivalent sub-expression for the original one in the original expression by **replacing the original sub-expression with a set of open parentheses**, and then afterwards **substituting the new equivalent sub-expression into the space between the parentheses** where the original sub-expression used to be.

First let's practice the first step, and then we will practice putting them all together.

Step 1: Identifying which PART of an expression has the same structure as an identity

Before we can use identities to rewrite part of an expression or equation, we have to practice identifying which parts of an expression have the structure of the right or the left side of the identity that we are interested in. Otherwise we have no idea how to use the identity to rewrite the expression.

For each of the following examples, simply identify one part of the expression that has the form of either the right or the left side of the given identity (there may be more than one correct answer).

Examples:

C) Circle some part of the expression below that has the structure of the left or the right side of the identity $\boldsymbol{a(b+c)=ab+ac}$ and identify exactly which parts of the expression would be equal to $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$:

$$\boldsymbol{(3x-2)+-3(x+7)}$$

Where in this expression can we see something that looks like either the left or right side of the identity above? The second half of this expression, $-3(x+7)$, has exactly the form of the left side of the identity, so we circle it:

$$(3x-2)+\boxed{-3(x+7)}$$

Now we identify which parts of this sub-expression $-3(x+7)$ correspond to a, b and c.
$a=-3$, because -3 is outside the parentheses to the left, and is being multiplied by the parentheses, just like a
$b=x$, because x is the first term inside the parentheses, just like b
$c=7$, because 7 is the second term inside the parentheses, just like c

D) Circle some part of the expression below that has the structure of the left or the right side of the identity $\boldsymbol{x^{-n}=\frac{1}{x^n}}$ (whenever $\boldsymbol{x\neq 0}$) and identify exactly which parts of the expression would be equal to $\boldsymbol{x}$ and $\boldsymbol{n}$:

$$\boldsymbol{5x(2x^2y)^{-3} \qquad (2x^2y\neq 0)}$$

Where in this expression can we see something that looks like either the left or the right side of the identity above? This part: $(2x^2y)^{-3}$ has exactly the form of the left side of the identity, so we circle this:
Now we identify which parts of this sub-expression $(2x^2y)^{-3}$ correspond to x and n.
$x=2x^2y$, because this is the base—the whole part that is being raised to the negative exponent
$n=3$, because this is the value of the exponent that comes after the negative sign

<u>Now you try!</u>

4. Circle some part of the expression below that has the structure of the left or the right side of the identity $x^{-n}=\frac{1}{x^n}$ (whenever $x\neq 0$) and **identify exactly which parts of the expression would be equal to $\boldsymbol{x}$ and $\boldsymbol{n}$**:

$$\frac{3x^4}{4x^2y^{-3}z^7}$$

5. Circle some part of the expression below that has the structure of the left or the right side of the identity $ba+ca=(b+c)a$ and **identify exactly which parts of the expression would be equal to $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$**:

$$4x^3+3x^2+8x^2+8x-1$$

6. Circle some part of the expression below that has the structure of the left or the right side of the identity $ab=ba$ and **identify exactly which parts of the expression would be equal to $\boldsymbol{a}$ and $\boldsymbol{b}$**. Choose <u>a different part</u> for <u>each</u> of the following criteria:

$$2x\cdot 3yx$$

   a. Choose anything you want for *a* and *b*, as long as the subexpression has the right structure.

   b. Choose something **different** for both *a* and *b* than what you chose in a) above.

   c. Choose something **more complex** for *a* or *b*, so that either *a* or *b* represent something that is a combination of more than one thing (e.g. a number and a variable, more than one variable, etc.).

7. Circle some part of the expression below that has the structure of the left or the right side of the identity $a - b = a + -b$ and **identify exactly which parts of the expression would be equal to $\boldsymbol{a}$ and $\boldsymbol{b}$**:
Be careful to consider the order of operations before making your choice, to determine which parts can be grouped together, and which cannot!

$$2x - 3x - 4 - x$$

   a. Choose anything you want for *a* and *b*, as long as the subexpression has the right structure.

   b. Choose something **different** for both *a* and *b* than what you chose in a) above.

   c. Choose something **more complex** for *a* or *b*, so that either *a* or *b* represent something that is a combination of more than one thing (e.g. a number and a variable, more than one variable, etc.).

All three steps: Using an identity to rewrite PART of an expression, and then substitute it back in

For each of the following examples, we use an identity to rewrite **part** of an expression or equation. To do this, we need to follow all three steps:

Step 1: Identify **which part** of the expression has the form of the left or right of the identity and circle that part.
Check to be sure that the part you have circled can be treated as a SINGLE UNIT, according to the conventions and the order of operations.

Step 2: Focus temporarily on rewriting **just that sub-expression** by using the identity to replace it with an equivalent sub-expression.

Step 3: **Substitute** the new equivalent sub-expression for the original one in the original expression by **replacing the original sub-expression with a set of open parentheses**, and then afterwards **substituting the new equivalent sub-expression into the space between the parentheses** where the original sub-expression used to be.

Examples:

E) Use the identity $\boldsymbol{a(b + c) = ab + ac}$ to replace the original expression with an equivalent expression that has a different form:

$$\boldsymbol{(3x - 2) + -3(x + 7)}$$

Where in this expression can we see something that looks like either the left or the right side of the identity above? The second half of this expression, $-3(x + 7)$, has exactly the form of the left side of the identity—this is the part that we can rewrite.
So for $-3(x + 7)$:
$a = -3,\ \ b = x,\ \ c = 7$

$$a(b + c) = ab + ac \quad \rightarrow \quad \overbrace{(\ \ )}^{a}\left(\overbrace{(\ \ )}^{b} + \overbrace{(\ \ )}^{c}\right) = \overbrace{(\ \ )}^{a}\overbrace{(\ \ )}^{b} + \overbrace{(\ \ )}^{a}\overbrace{(\ \ )}^{c}$$

If we just wanted to rewrite $-3(x + 7)$, we would write only this: $\overbrace{(-3)}^{a}\left(\overbrace{(x)}^{b} + \overbrace{(7)}^{c}\right) = \overbrace{(-3)}^{a}\overbrace{(x)}^{b} + \overbrace{(-3)}^{a}\overbrace{(7)}^{c}$

However, we need to rewrite the **whole** original expression, so we can't leave out the first term. So once we have figured out how to rewrite $-3(x + 7)$, we have to go back to the original expression like this:
$(3x - 2) + \boxed{(-3(x + 7))} = (3x - 2) + \boxed{\big((-3)(x) + (-3)(7)\big)}$

So $\boxed{\boldsymbol{(3x - 2) + -3(x + 7) = (3x - 2) + \big((-3)(x) + (-3)(7)\big)}}$

F) Use the identity $x^{-n} = \frac{1}{x^n}$ (whenever $x \neq 0$) to replace the original expression with an equivalent expression that has a different form:

$$\mathbf{5x(2x^2y)^{-3} \qquad (2x^2y \neq 0)}$$

This part: $(2x^2y)^{-3}$ has exactly the form of the left side of the identity—this is the part of we can rewrite.
So for $(2x^2y)^{-3}$:
$x = 2x^2y, \ \ n = 3$

$$\frac{1}{x^n} = x^{-n} \quad \rightarrow \quad \overbrace{(\ \ )}^{x}{}^{-\overbrace{(\ \ )}^{n}} = \frac{1}{\underbrace{(\ \ )}_{x}{}^{\overbrace{(\ \ )}^{n}}}$$

$$\overbrace{(2x^2y)}^{x}{}^{-\overbrace{(3)}^{n}} = \frac{1}{\overbrace{(2x^2y)}^{x}{}^{\overbrace{(3)}^{n}}}$$ (We know that $x \neq 0$ because $2x^2y \neq 0$.)

If we just wanted to rewrite $(2x^2y)^{-3}$, we would write only this: $\overbrace{(2x^2y)}^{x}{}^{-\overbrace{(3)}^{n}} = \frac{1}{\overbrace{(2x^2y)}^{x}{}^{\overbrace{(3)}^{n}}}$

However, we need to rewrite the **whole** original expression, so we can't leave out the first term. So once we have figured out how to rewrite $(2x^2y)^{-3}$, we have to go back to the original expression like this:

$$5x\boxed{((2x^2y)^{-3})} = 5x\boxed{\left(\frac{1}{\overbrace{(2x^2y)}^{x}{}^{\overbrace{(3)}^{n}}}\right)}$$

So $$\boxed{\mathbf{5x(2x^2y)^{-3} = 5x\left(\frac{1}{\overbrace{(2x^2y)}^{x}{}^{\overbrace{(3)}^{n}}}\right)}}$$

<u>**Now you try!** For each of the following problems, do ALL of the following steps:</u>

A) **CIRCLE the sub-expression** that has the form of either the left or right of the identity (**check to be sure** that you can treat that sub-expression as a single unit based on the conventions and the order of operations), then
B) **write out what the various variables in the identity would have to be equal to** in order to use the identity to replace that sub-expression with an equivalent one with a different form, then
C) **use the identity to replace that sub-expression with an equivalent one** with a different form, and finally
D) **substitute the new sub-expression in for the original one**—be sure to **USE PARENTHESES in place of the original sub-expression**, and to **leave all of the expression structure outside the parentheses unchanged**.

8. Use the identity $x^{-n} = \frac{1}{x^n}$ (whenever $x \neq 0$) to replace the original expression with an equivalent expression that has a different form:

$$\frac{3x^4}{4x^2y^{-3}z^7}$$

9. Use the identity $ba + ca = (b + c)a$ to replace the original expression with an equivalent expression that has a different form:

$$4x^3 + 3x^2 + 8x^2 + 8x - 1$$

10. Use the identity $ab = ba$ to replace the original expression with an equivalent expression that has a different form. (Extra credit if you can do this in such a way that all the numbers in the new expression end up next to each other and all the same variables end up next to each other. You will need to use this identity in more than one place to accomplish this):

$$2x \cdot 3yx$$

11. Use the identity $x = 1x$ to replace the original expression with an equivalent expression that has a different form—specifically, use this identity to rewrite the first x^2 in this expression:

$$\sqrt{x^2 + 3x^2}$$